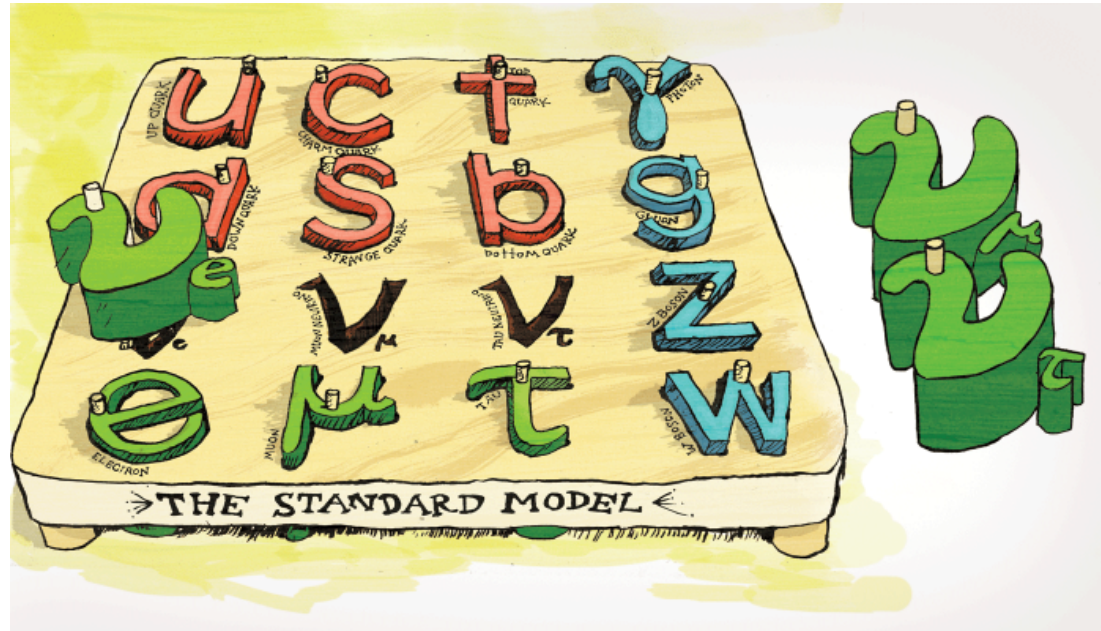


Three-Flavor Paradigm, Overview



*Snowmass Neutrino Colloquium I:
Neutrinos and the Three-Flavor Paradigm*

André de Gouvêa – Northwestern University

Something Funny Happened on the Way to the 21st Century^a

ν Flavor Oscillations

Neutrino oscillation experiments have revealed that **neutrinos change flavor** after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L . The evidence is overwhelming.

- $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ — atmospheric and accelerator experiments;
- $\nu_e \rightarrow \nu_{\mu,\tau}$ — solar experiments;
- $\bar{\nu}_e \rightarrow \bar{\nu}_{\text{other}}$ — reactor experiments;
- $\nu_\mu \rightarrow \nu_{\text{other}}$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_{\text{other}}$ — atmospheric and accelerator expts;
- $\nu_\mu \rightarrow \nu_e$ — accelerator experiments.

The simplest and **only satisfactory** explanation of **all** these data is that neutrinos have distinct masses, and mix.

^a2002+2015 Physics Nobel Prizes, 2016 Breakthrough Prize in Fundamental Physics.

A Realistic, Reasonable, and Simple Paradigm:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{e\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3):

- $m_1^2 < m_2^2$ $\Delta m_{31}^2 < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ $\Delta m_{31}^2 > 0$ – Normal Mass Hierarchy

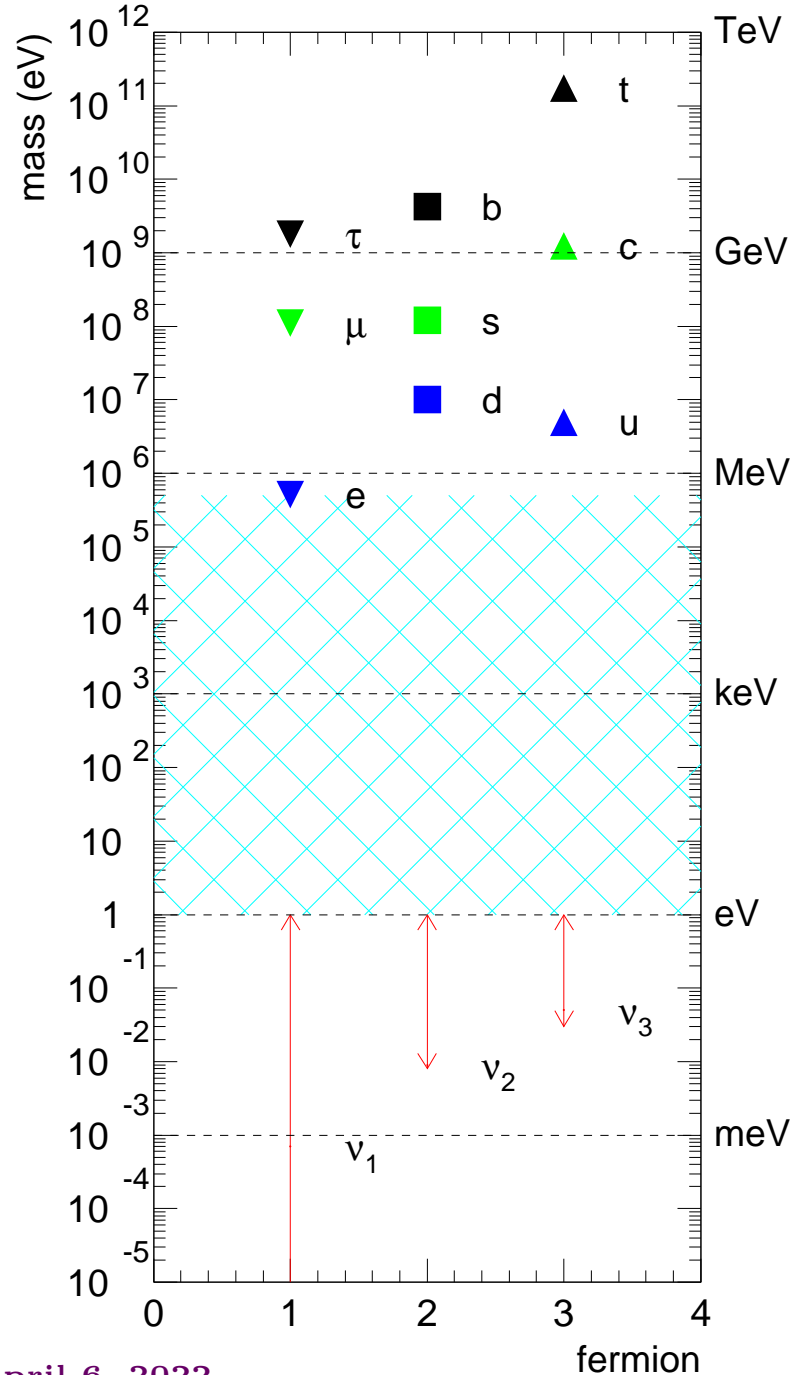
$$\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}; \quad U_{e3} \equiv \sin \theta_{13} e^{-i\delta}$$

This Standard Three-Massive-Active Neutrinos Paradigm fits, for the most part, all data very well^a

Furthermore, most of the oscillation parameters have been measured quite precisely: (see, for example, <http://www.nu-fit.org>)

$$\begin{aligned}
 \Delta m_{21}^2 &= (7.42 \pm 0.21) \times 10^{-5} \text{ eV}^2 & (3\%) \\
 |\Delta m_{31}^2| &= (2.50 \pm 0.03) \times 10^{-3} \text{ eV}^2 & (1\%) \\
 \sin^2 \theta_{12} &= 0.304 \pm 0.013 & (4\%) \\
 \sin^2 \theta_{13} &= 0.02220 \pm 0.00068 & (3\%) \\
 \sin^2 \theta_{23} &= 0.573 \pm 0.023 & (5\%) \\
 \delta_{CP} &= (105 - 405)^\circ (3\sigma) & (\text{unknown}) \\
 \text{sign}(\Delta m_{31}^2) &= +, \text{ slightly favored} & (\text{unknown})
 \end{aligned} \tag{1}$$

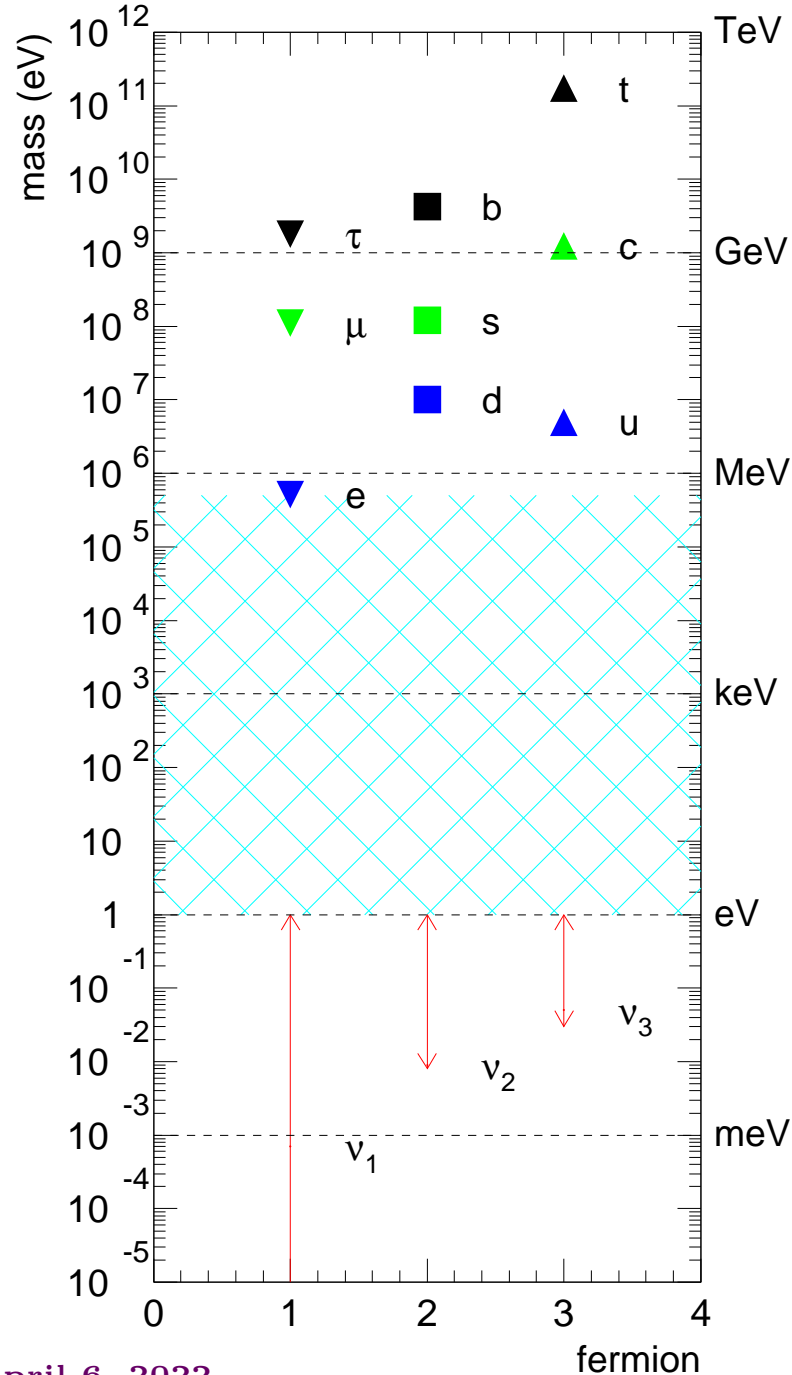
^aModulo the short-baseline anomalies which I will not discuss.



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?



NEW PHYSICS

Nonzero neutrino masses imply the existence of new fundamental fields \Rightarrow **New Particles**

We know nothing about these new particles. They can be **bosons** or **fermions**, very light or very heavy, they can be charged or neutral, experimentally accessible or hopelessly out of reach...

There is only a handful of questions the standard model for particle physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there so much ordinary matter in the Universe? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).

Neutrino Masses, Higgs Mechanism, and New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs doublet model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly**. And **lepton-number must be an exact symmetry** of nature (or broken very, very weakly);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking!;
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the **seesaw mechanism**.

We are going to need a lot of experimental information from all areas of particle physics in order to figure out what is really going on!

Piecing the Neutrino Mass Puzzle

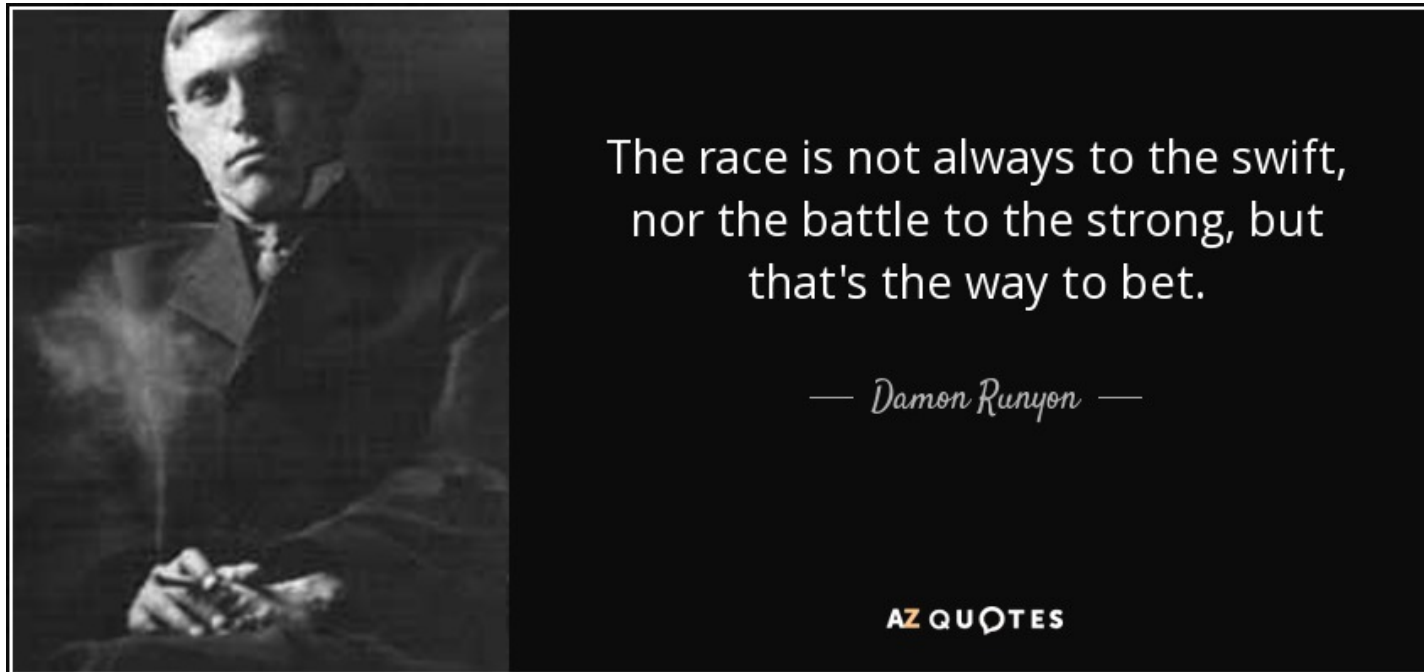
Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts . . .

- understanding the fate of lepton-number. Neutrinoless double-beta decay.
- **A comprehensive long baseline neutrino program.**
- Probes of neutrino properties, including neutrino scattering experiments. And what are the neutrino masses anyway? Kinematical probes.
- Precision measurements of charged-lepton properties ($g - 2$, edm) and searches for rare processes ($\mu \rightarrow e$ -conversion the best bet at the moment).
- Collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- Neutrino properties affect, in a significant way, the history of the universe. These can be “seen” in cosmic surveys of all types.
- Astrophysical Neutrinos – Supernovae and other Galaxy-shattering phenomena. Ultra-high energy neutrinos and correlations with not-neutrino messengers.

HOWEVER...

We have only ever objectively “seen” neutrino masses in long-baseline oscillation experiments. It is one unambiguous way forward!

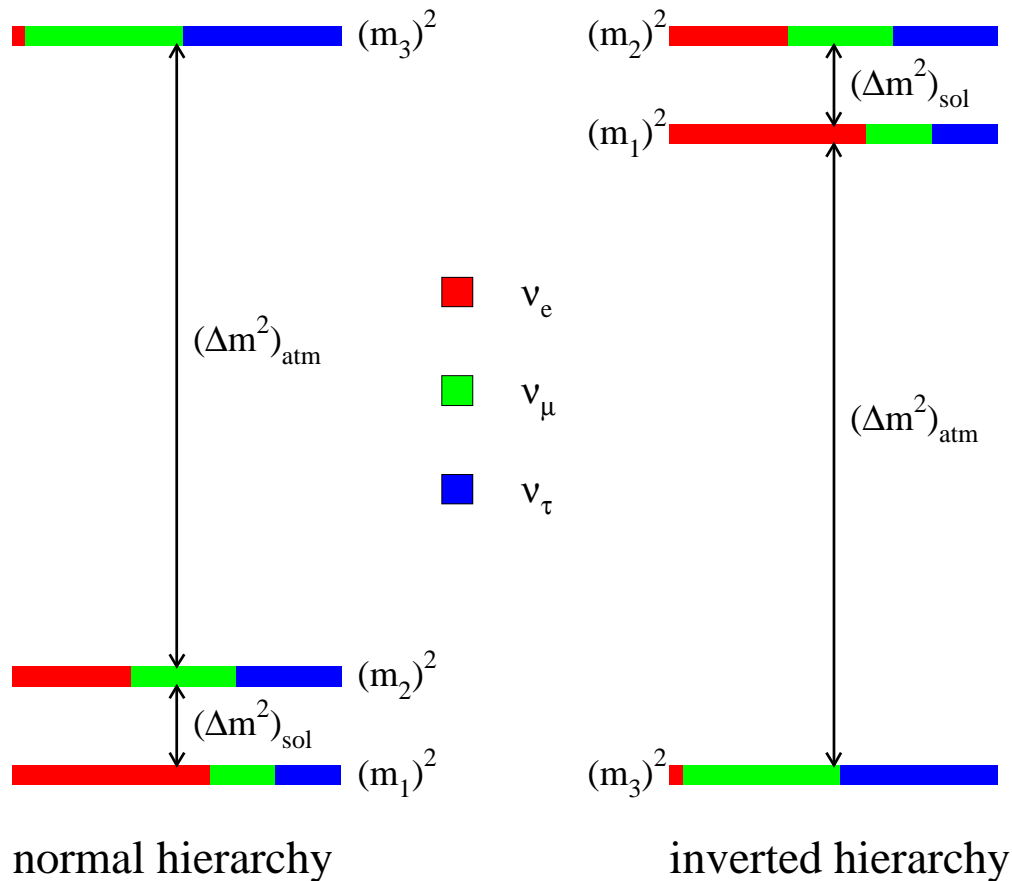
Does this mean we will reveal the origin of neutrino masses with oscillation experiments? We don't know, and we won't know until we try!



Long-Baseline Experiments, Present and Future (Not Exhaustive!)

- [NOW] T2K (Japan), NO ν A (USA) – $\nu_\mu \rightarrow \nu_e$ appearance, ν_μ disappearance – precision measurements of “atmospheric parameters” ($\Delta m_{31}^2, \sin^2 \theta_{23}$). Pursue mass hierarchy via matter effects. Nontrivial tests of paradigm. First step towards CP-invariance violation.
- [SOON] JUNO (China) – $\bar{\nu}_e$ disappearance – precision measurements of “solar parameters” ($\Delta m_{12}^2, \sin^2 \theta_{12}$). Pursue the mass hierarchy via precision measurements of oscillations.
- [SOON] km³ arrays, upgraded – atmospheric neutrinos – pursue mass hierarchy via matter effects.
- [LATER] HyperK (Japan), DUNE (USA) – Second step towards CP-invariance violation. More nontrivial tests of the paradigm. Ultimate “super-beam” experiments.

Missing Oscillation Parameters: Are We There Yet? (NO!)

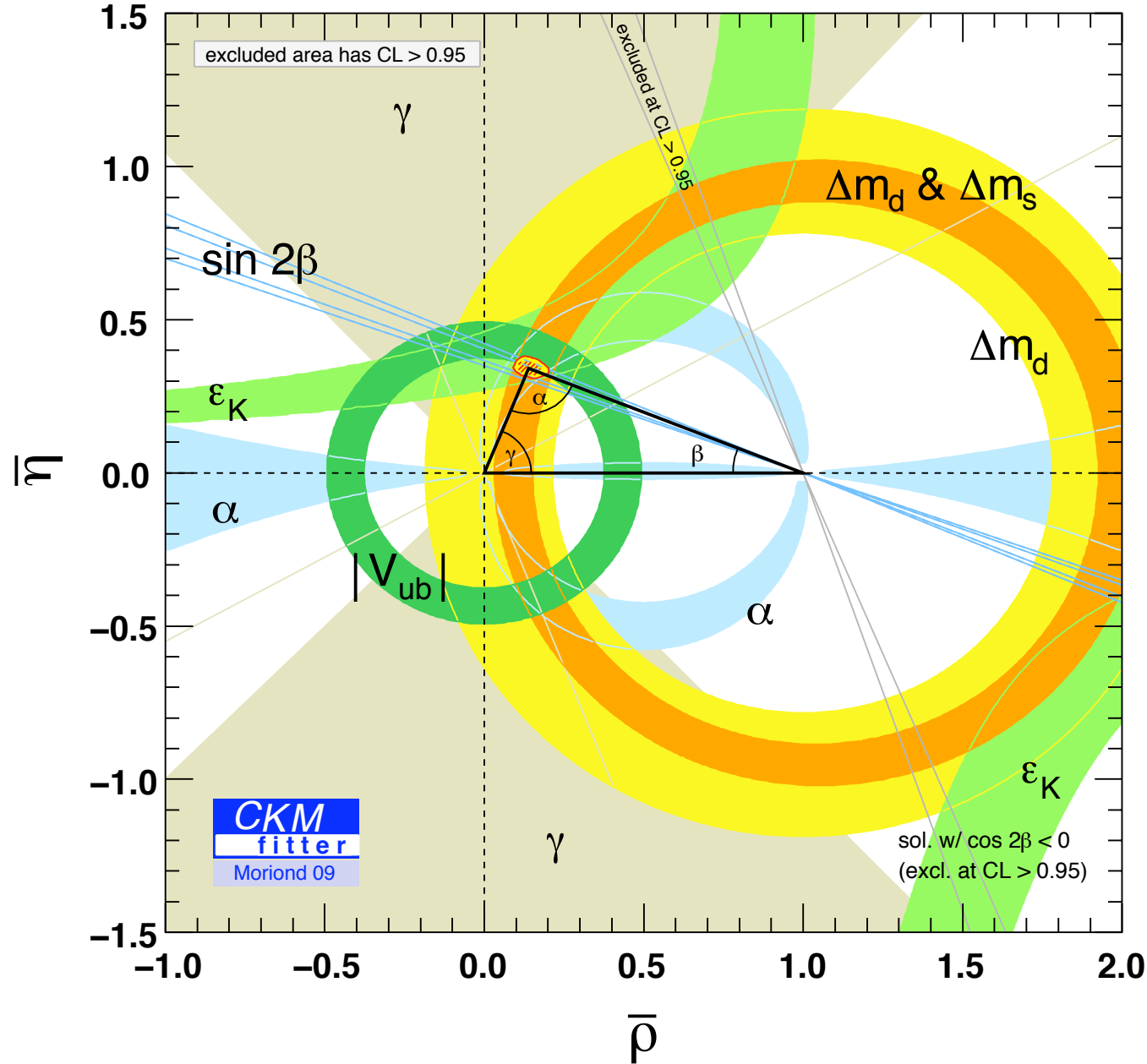


- ~~What is the ν_e component of ν_3 ? ($\theta_{13} \neq 0$!)~~
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m_{13}^2 > 0$?)

\Rightarrow All of the above can “only” be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)

What we ultimately want to achieve:



We need to do this in the lepton sector!

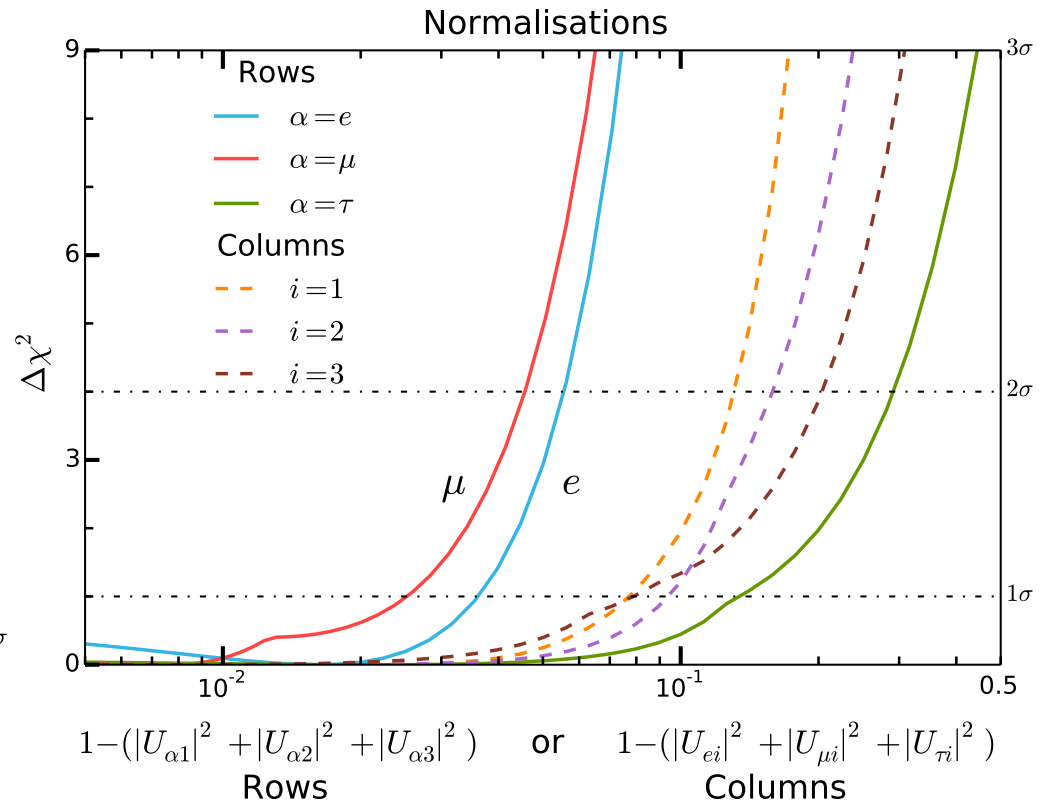
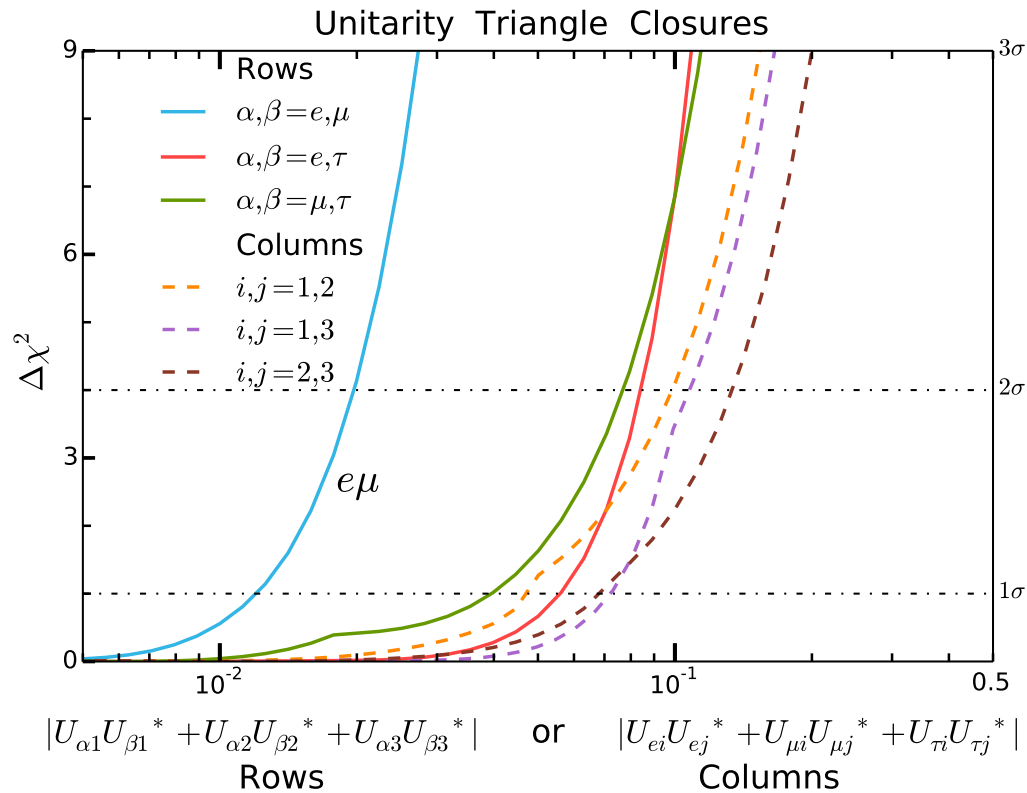
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

What we have **really measured** (very roughly):

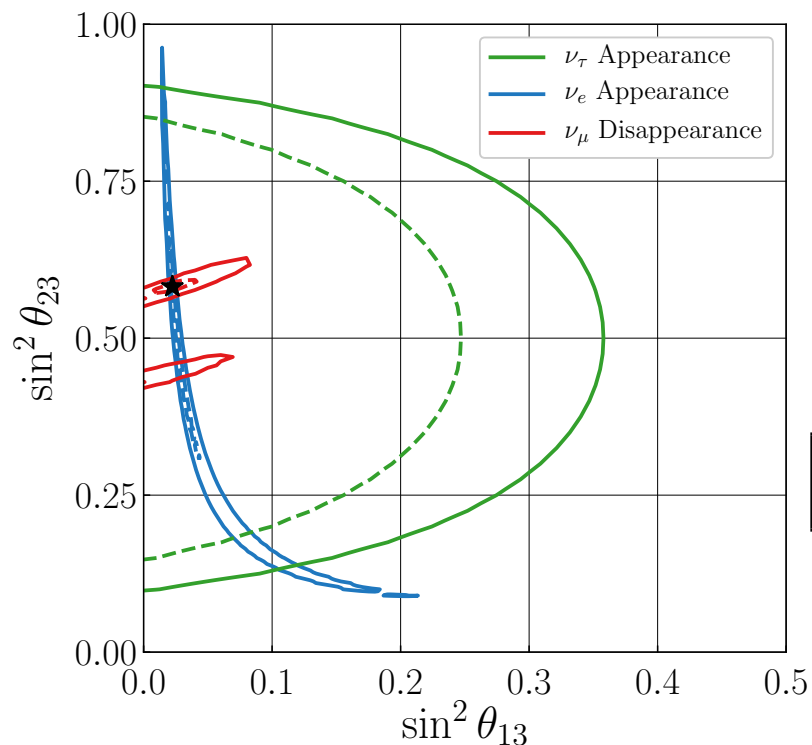
- Two mass-squared differences – many probes;
- $|U_{e2}|^2$ – solar data;
- $|U_{\mu2}|^2 + |U_{\tau2}|^2$ – solar data;
- $|U_{e2}|^2|U_{e1}|^2$ – KamLAND;
- $|U_{\mu3}|^2(1 - |U_{\mu3}|^2)$ – atmospheric data, long-baseline accelerator experiments;
- $|U_{e3}|^2(1 - |U_{e3}|^2)$ – Double Chooz, Daya Bay, RENO;
- $|U_{\mu3}|^2|U_{\tau3}|^2$ – atmospheric, OPERA;
- $|U_{e3}|^2|U_{\mu3}|^2$ – NOvA, T2K.

We still have a long way to go!

A little more quantitative:



[Parke and Ross-Lonergan, arXiv:1508.05095]



DUNE 7 yr. data collection

3.5 yr. Neutrino Mode, 3.5 yr. Antineutrino Mode

$\sin^2 \theta_{12} = 0.310$ (fixed)

$\sin^2 \theta_{13} = 0.02240$ (free)

$\sin^2 \theta_{23} = 0.582$ (free)

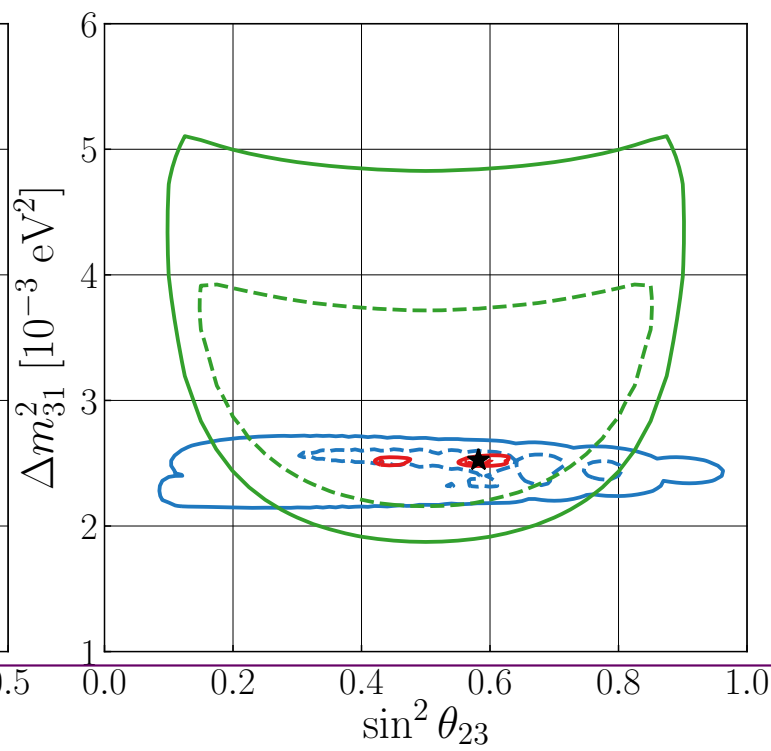
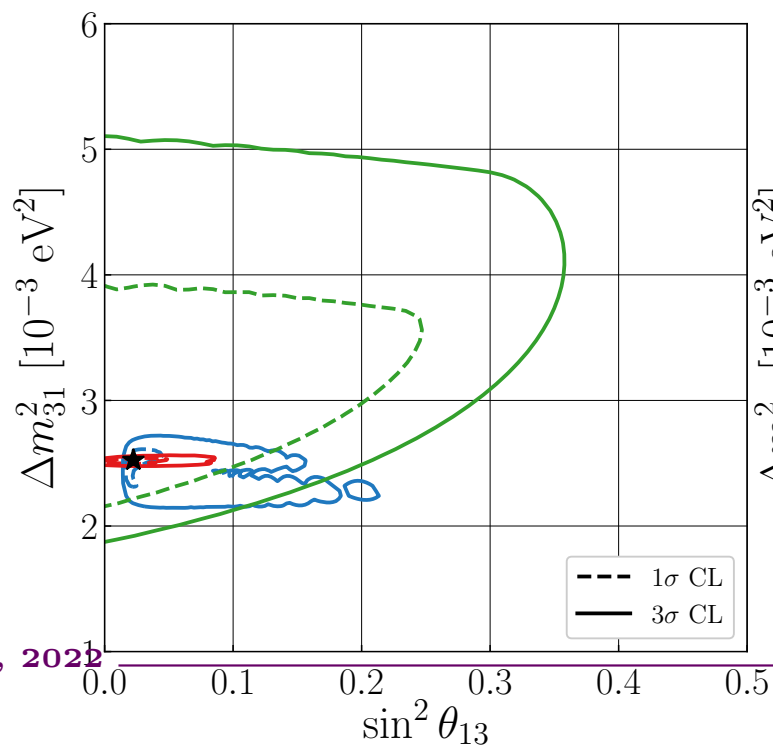
$\Delta m_{21}^2 = 7.39 \times 10^{-5} \text{ eV}^2$ (fixed)

$\Delta m_{31}^2 = +2.525 \times 10^{-3} \text{ eV}^2$ (free, ordering fixed)

$\delta_{CP} = -2.496 \text{ rad} = 217^\circ$ (free)

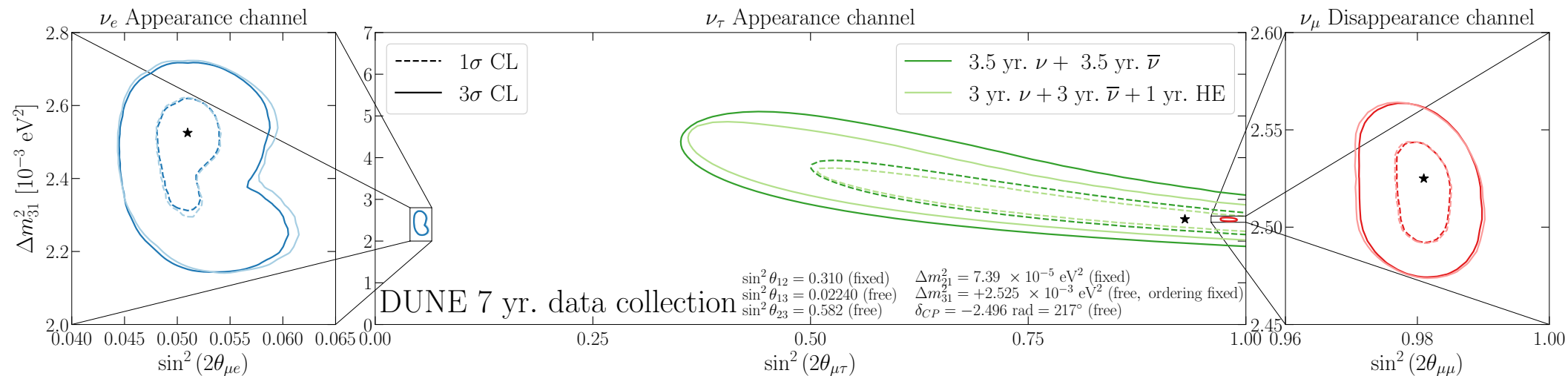
Testing the Three-Neutrinos Paradigm

[AdG, Kelly, Pasquini, Stenico, arXiv:1904.07265]



Unitarity test with DUNE, including ν_τ appearance

[AdG, Kelly, Pasquini, Stenico, arXiv:1904.07265]



$$\sin^2 2\theta_{\mu e} \equiv 4|U_{\mu 3}|^2|U_{e 3}|^2, \quad \sin^2 2\theta_{\mu \tau} \equiv 4|U_{\mu 3}|^2|U_{\tau 3}|^2, \quad \sin^2 2\theta_{\mu \mu} \equiv 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)$$

(Warning: Busy plot. the x -axes are different for each of the three different countours!)

$$\text{Unitarity Test: } |U_{e 3}|^2 + |U_{\mu 3}|^2 + |U_{\tau 3}|^2 = 1_{-0.06}^{+0.05} \text{ [one sigma]} \quad (1_{-0.17}^{+0.13} \text{ [three sigma]})$$

What Could We Run Into?



since $m_\nu \neq 0$ and leptons mix ...

What Could We Run Into?

- New neutrino states. In this case, the 3×3 mixing matrix would not be unitary.
- New short-range neutrino interactions. These lead to, for example, new matter effects. If we don't take these into account, there is no reason for the three flavor paradigm to “close.”
- New, unexpected neutrino properties. Do they have nonzero magnetic moments? Do they decay? The answer is ‘yes’ to both, but nature might deviate dramatically from ν SM expectations.
- Weird stuff. CPT-violation. Decoherence effects (aka “violations of Quantum Mechanics.”)
- etc.

Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates **five** irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don't understand its value. At all.
- One is θ_{QCD} term ($\theta G\tilde{G}$). We don't know its value but it is only constrained to be very small. We don't know why (there are some good ideas, however).
- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don't know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why? Cautionary tale: “Mixing angles are small.”

Indirect connection to the matter–antimatter asymmetry of the universe. The existence of new sources of CP-invariance violation is a necessary requirement.

Understanding Fermion Mixing

One of the puzzling phenomena uncovered by the neutrino data is the fact that **Neutrino Mixing is Strange**. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \quad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix} \quad \boxed{\text{WHY?}}$$

(They certainly look VERY different, but which one would you label as “strange”?)

Precision Meas. of Oscillation Parameters. Why and How Much?

A word from flavor models:

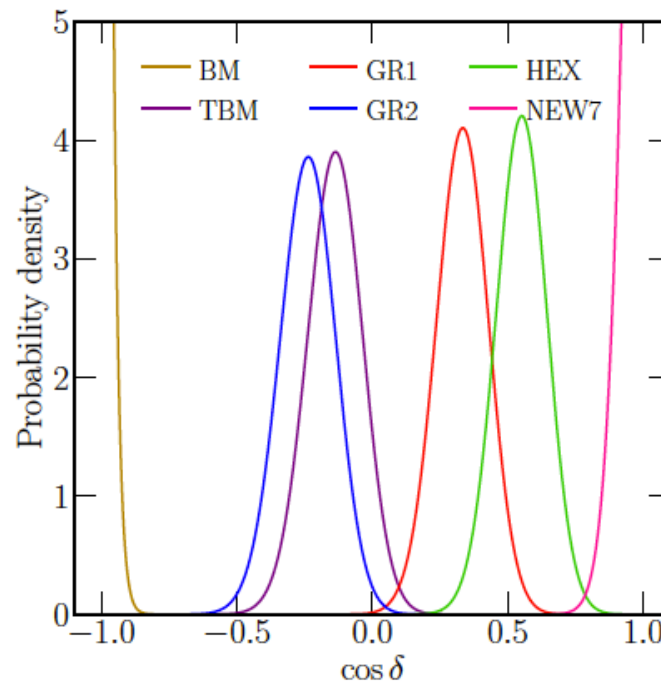


Figure 2: $P_{\cos \delta}$ as a function of $\cos \delta$ for various mixing patterns. Here we have assumed that $P_z(z)$ is a Gaussian centered at the experimental best-fit value of z , with width of 1σ .

[Everett *et al.*, arXiv:1912.10139]

More General Comments.

If there is an underlying structure behind the values of the lepton masses and mixing angles...

- it may lead to relations among the parameters: **sum rules**.

$$f(\theta_{12}, \theta_{13}, \theta_{23}, \delta, m_1, m_2, m_3) = 0.$$

- it may lead to relations between PMNS and CKM parameters.

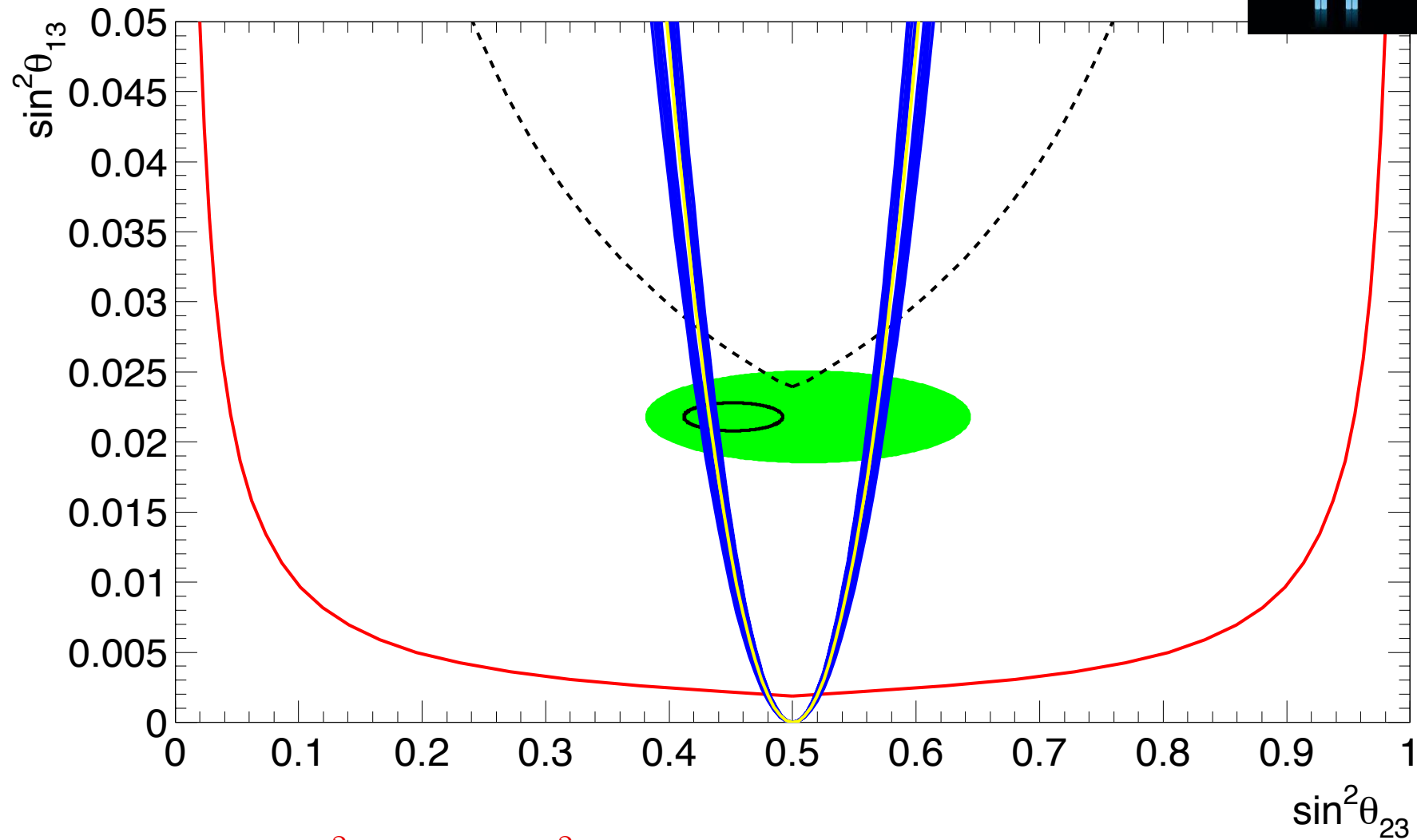
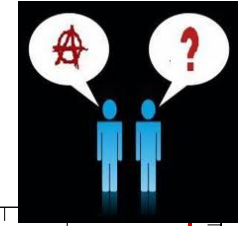
$$f(\text{PMNS}) = g(\text{CKM}).$$

- etc.

These provide guidance for precision.

- Sum rules need all oscillation parameters to be known with similar precision: θ_{23}, δ are the obvious outliers.
- On the CKM side, $\theta_{12} = 13.04^\circ \pm 0.05^\circ$, $\theta_{13} = 0.201^\circ \pm 0.011^\circ$, $\theta_{23} = 2.38^\circ \pm 0.06^\circ$, $\delta = 68.8^\circ \pm 4.5^\circ$. (several percent to sub percent).

Anarchy vs. Order — more precision required!



Order: $\sin^2 \theta_{13} = C \cos^2 2\theta_{23}$, $C \in [0.8, 1.2]$

[AdG, Murayama, 1204.1249]

In conclusion...

- We still **know very little** about the new physics uncovered by neutrino oscillations.
- **neutrino masses are very small** – we don't know why, but we think it means something important.
- **neutrino mixing is “weird”** – we don't know why, but we think it means something important.
- We **need more experimental input** (neutrinoless double-beta decay, precision neutrino oscillations, UHE neutrinos, charged-lepton precision measurements, colliders, etc).
- **Precision measurements of neutrino oscillations** are sensitive to several new phenomena, including new neutrino properties, the existence of new states, or the existence of new interactions.
- There is plenty of **room for surprises**, as neutrinos are potentially very deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are “quantum interference devices” – potentially very sensitive to whatever else may be out there (e.g., $\Lambda \simeq 10^{14}$ GeV).

Backup Slides . . .



Case Studies

I will discuss a few case-studies, including the **fourth-neutrino hypothesis** and **non-standard neutral-current neutrino–matter interactions**. In general

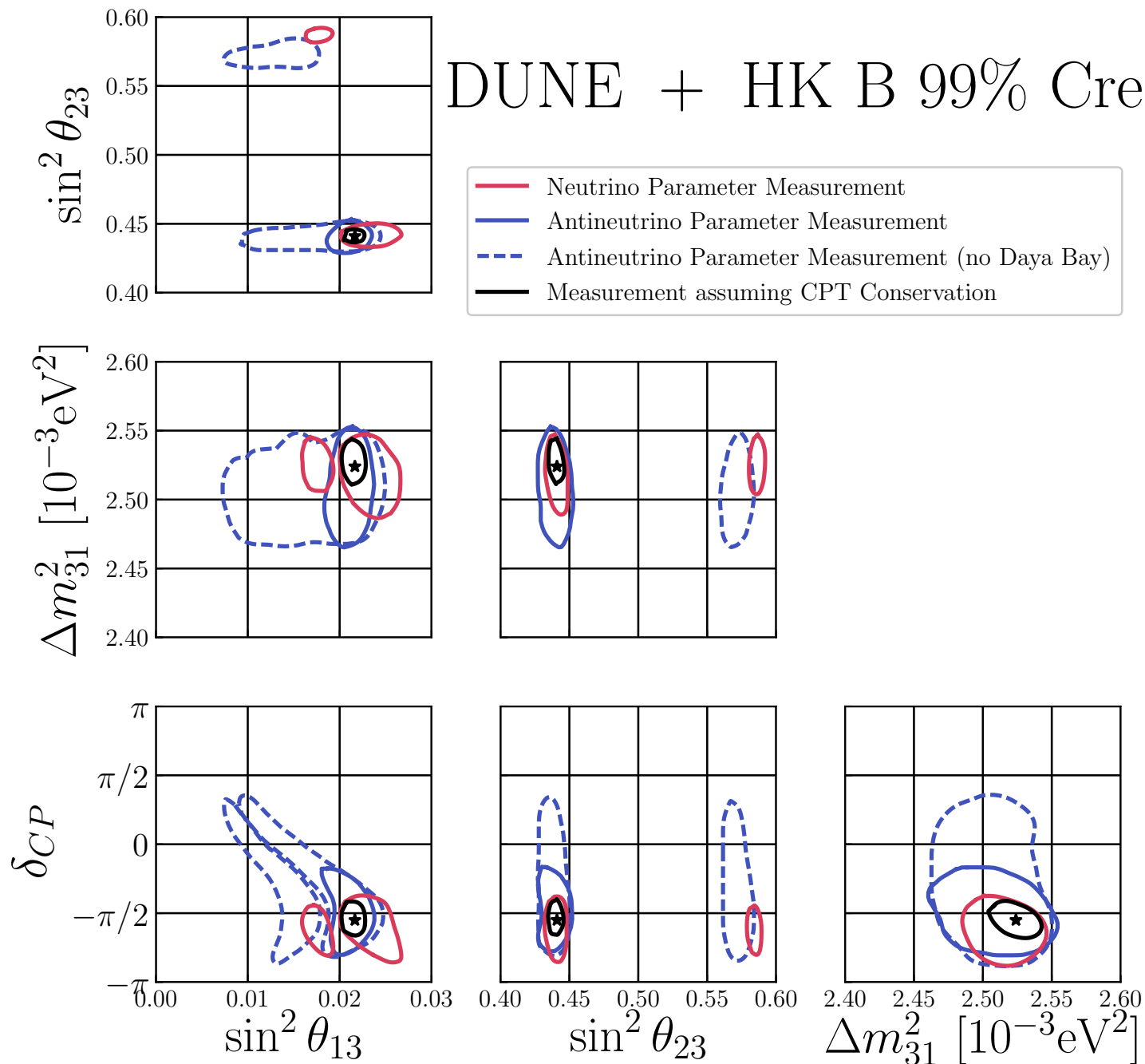
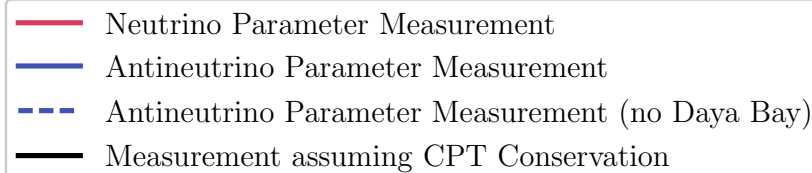
- I will mostly discuss, for concreteness, the DUNE setup;
- I don't particularly care about how likely, nice, or contrived the scenarios are. It is useful to consider them as well-defined ways in which the three-flavor paradigm can be violated. They can be used as benchmarks for comparing different efforts, or, perhaps, as proxies for other new phenomena.
- I will mostly be interested in three questions:
 - How sensitive are next-generation long-baseline efforts?;
 - How well they can measure the new-physics parameters, including new sources of CP-invariance violation?;
 - Can they tell different new-physics models apart?

Different Oscillation Parameters for Neutrinos and Antineutrinos?

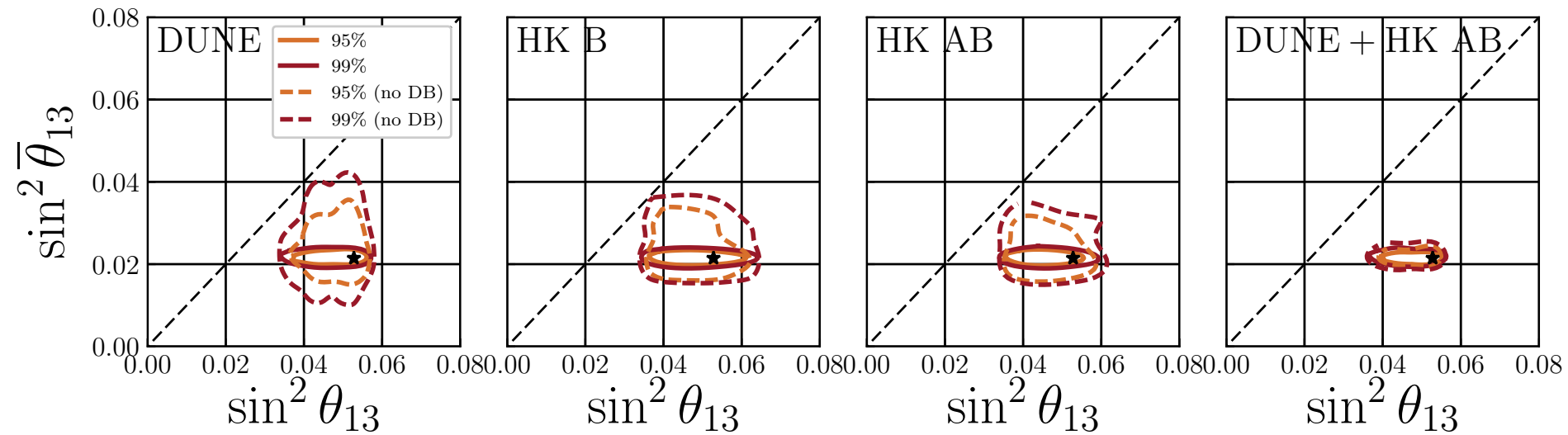
[AdG, Kelly, arXiv:1709.06090]

- How much do we know, independently, about neutrino and antineutrino oscillations?
- What happens if the parameters disagree?

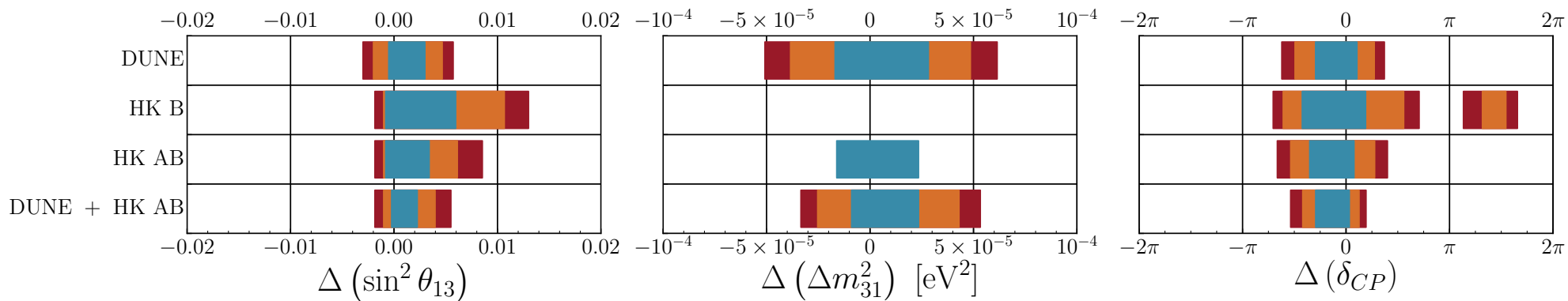
DUNE + HK B 99% Cred.



[AdG and Kelly, arXiv:1709.06090]



[AdG and Kelly, arXiv:1709.06090]



A Fourth Neutrino

(Berryman et al, arXiv:1507.03986)

If there are more neutrinos with a well-defined mass, it is easy to extend the paradigm:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_? \\ \vdots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \cdots \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \cdots \\ U_{?1} & U_{?2} & U_{?3} & U_{?4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix}$$

- New mass eigenstates easy: ν_4 with mass m_4 , ν_5 with mass m_5 , etc.
- What are these new “flavor” (or weak) eigenstates $\nu_?$? Here, the answer is we don’t care. We only assume there are no new accessible interactions associated to these states.

$$\begin{aligned}
U_{e2} &= s_{12}c_{13}c_{14}, \\
U_{e3} &= e^{-i\eta_1} s_{13}c_{14}, \\
U_{e4} &= e^{-i\eta_2} s_{14}, \\
U_{\mu 2} &= c_{24} (c_{12}c_{23} - e^{i\eta_1} s_{12}s_{13}s_{23}) - e^{i(\eta_2-\eta_3)} s_{12}s_{14}s_{24}c_{13}, \\
U_{\mu 3} &= s_{23}c_{13}c_{24} - e^{i(\eta_2-\eta_3-\eta_1)} s_{13}s_{14}s_{24}, \\
U_{\mu 4} &= e^{-i\eta_3} s_{24}c_{14}, \\
U_{\tau 2} &= c_{34} (-c_{12}s_{23} - e^{i\eta_1} s_{12}s_{13}c_{23}) - e^{i\eta_2} c_{13}c_{24}s_{12}s_{14}s_{34} \\
&\quad - e^{i\eta_3} (c_{12}c_{23} - e^{i\eta_1} s_{12}s_{13}s_{23}) s_{24}s_{34}, \\
U_{\tau 3} &= c_{13}c_{23}c_{34} - e^{i(\eta_2-\eta_1)} s_{13}s_{14}s_{34}c_{24} - e^{i\eta_3} s_{23}s_{24}s_{34}c_{13}, \\
U_{\tau 4} &= s_{34}c_{14}c_{24}.
\end{aligned}$$

When the new mixing angles ϕ_{14} , ϕ_{24} , and ϕ_{34} vanish, one encounters oscillations among only three neutrinos, and we can map the remaining parameters $\{\phi_{12}, \phi_{13}, \phi_{23}, \eta_1\} \rightarrow \{\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}\}$.

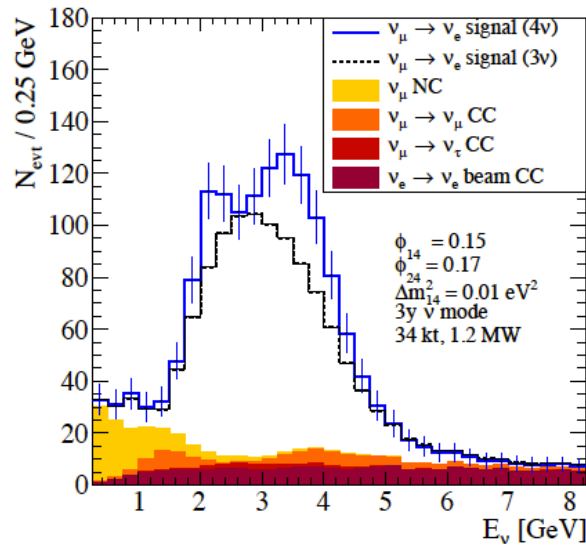
Also

$$\eta_s \equiv \eta_2 - \eta_3,$$

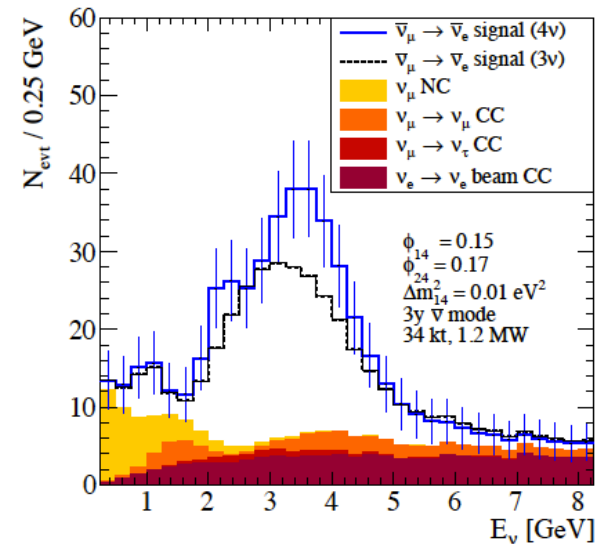
is the only new CP-odd parameter to which oscillations among ν_e and ν_μ are sensitive.

Some technicalities for the aficionados

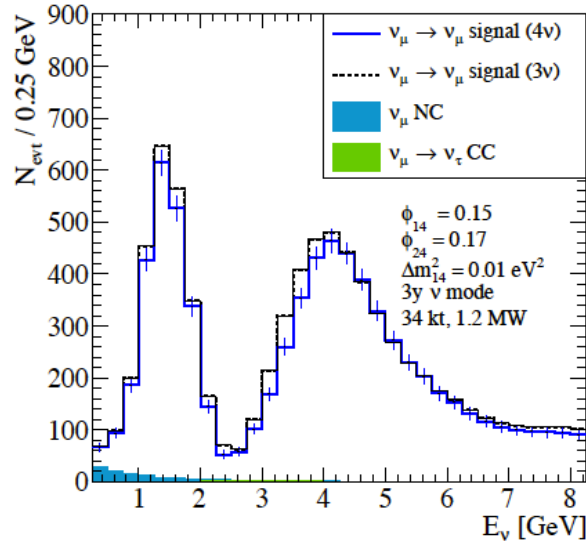
- 34 kiloton liquid argon detector;
- 1.2 MW proton beam on target as the source of the neutrino and antineutrino beams, originating 1300 km upstream at Fermilab;
- 3 years each with the neutrino and antineutrino mode;
- Include standard backgrounds, and assume a 5% normalization uncertainty;
- Whenever quoting bounds or measurements of anything, we marginalize over all parameters not under consideration;
- We include priors on Δm_{12}^2 and $|U_{e2}|^2$ in order to take into account information from solar experiments and KamLAND. Unless otherwise noted, we assume the mass ordering is normal;
- We do not include information from past experiments. We assume that DUNE will “out measure” all experiments that came before it (except for the solar ones, as mentioned above).



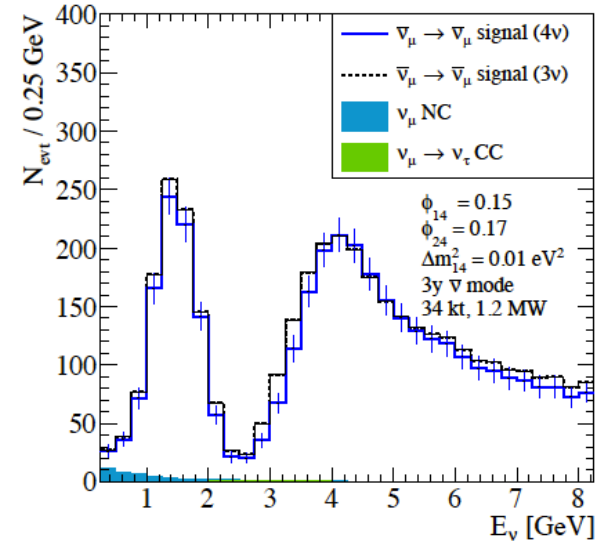
(a)



(b)



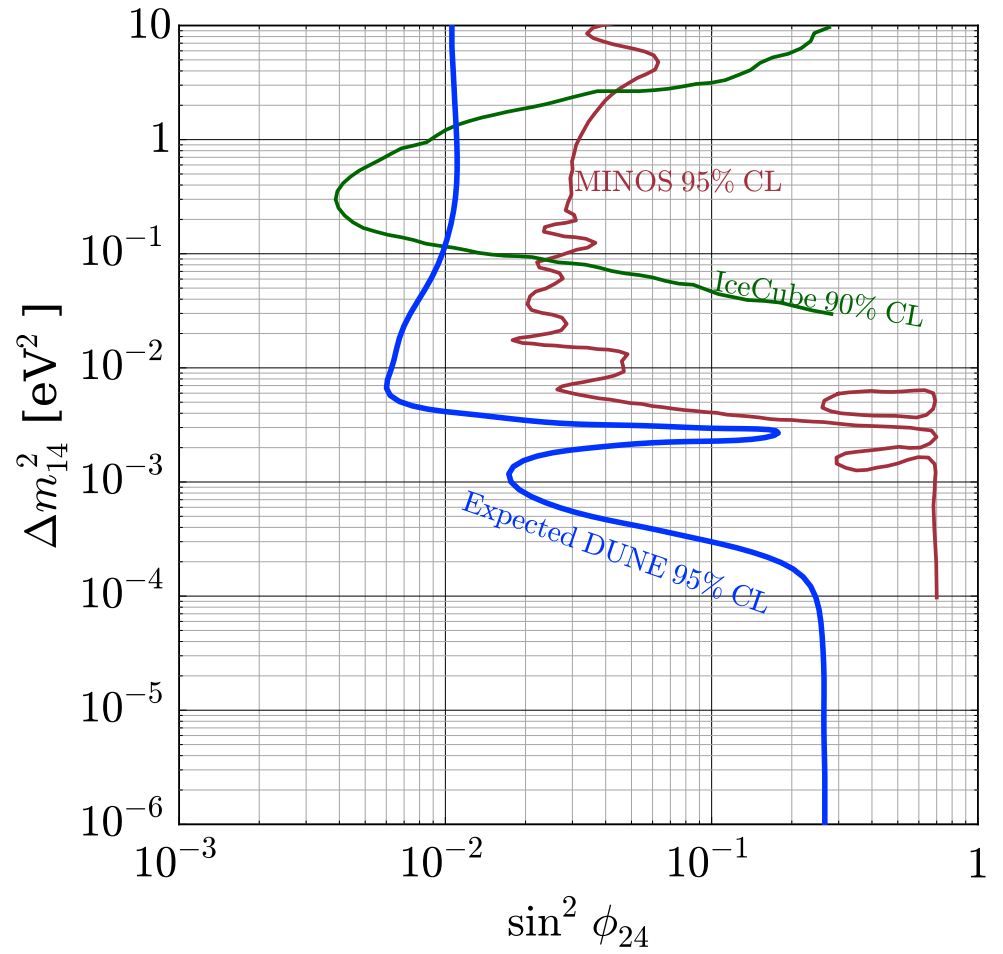
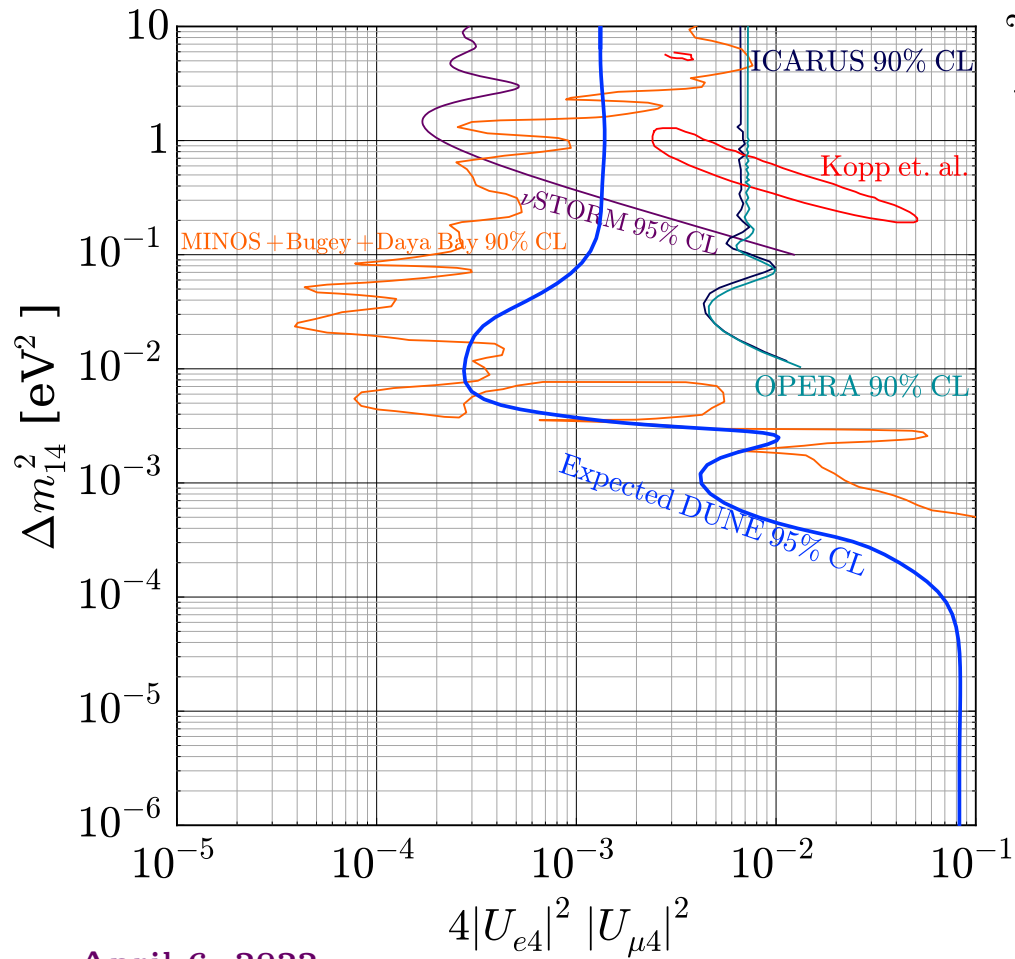
(c)



(d)

[Berryman et al, arXiv:1507.03986]

FIG. 1: Expected signal and background yields for six years (3y ν + 3y $\bar{\nu}$) of data collection at DUNE, using fluxes projected by Ref. [1], for a 34 kiloton detector, and a 1.2 MW beam. (a) and (b) show appearance channel yields for neutrino and antineutrino beams, respectively, while (c) and (d) show disappearance channel yields. The 3 ν signal corresponds to the standard three-neutrino hypothesis, where $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{13} = 0.0235$, $\sin^2 \theta_{23} = 0.437$, $\Delta m_{12}^2 = 7.54 \times 10^{-5} \text{ eV}^2$, $\Delta m_{13}^2 = 2.43 \times 10^{-3} \text{ eV}^2$, $\delta_{CP} = 0$, while the 4 ν signal corresponds to $\sin^2 \phi_{12} = 0.315$, $\sin^2 \phi_{13} = 0.024$, $\sin^2 \phi_{23} = 0.456$, $\sin^2 \phi_{14} = 0.023$, $\sin^2 \phi_{24} = 0.030$, $\Delta m_{14}^2 = 10^{-2} \text{ eV}^2$, $\eta_1 = 0$, and $\eta_s = 0$. Statistical uncertainties are shown as vertical bars in each bin. Backgrounds are defined in the text and are assumed to be identical for the three- and four-neutrino scenarios: any discrepancy is negligible after accounting for a 5% normalization uncertainty.



[Berryman et al, arXiv:1507.03986]

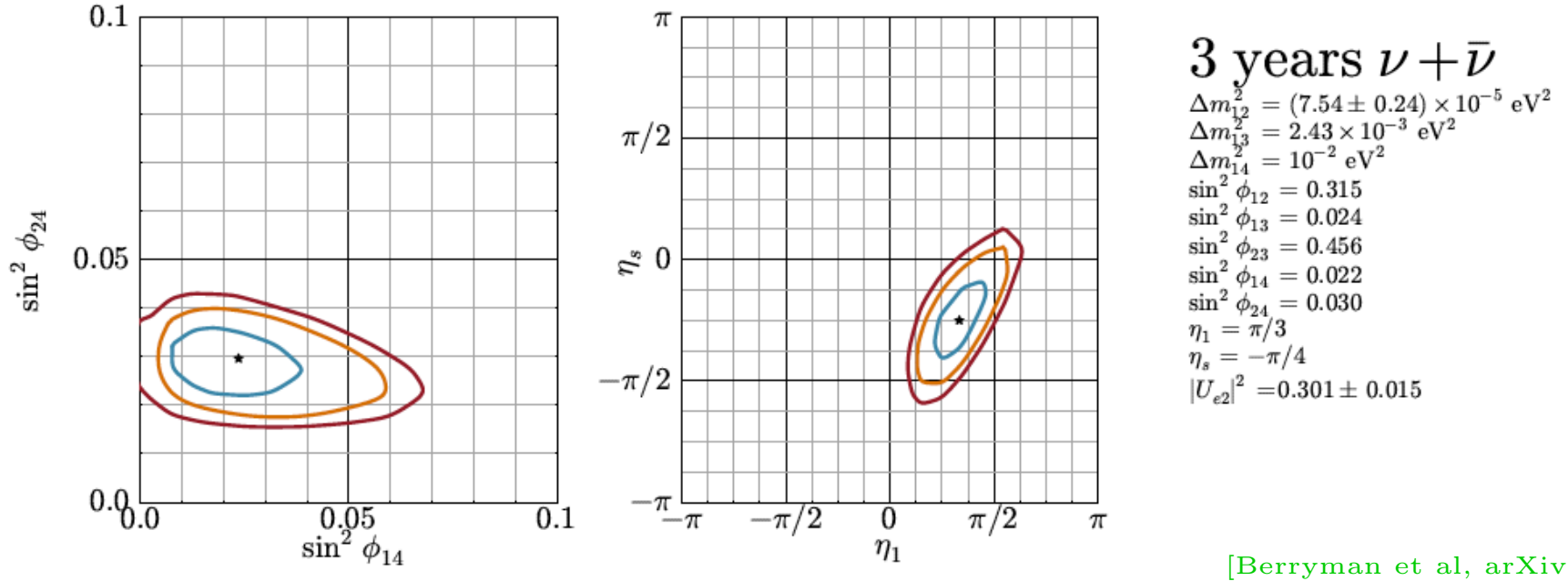
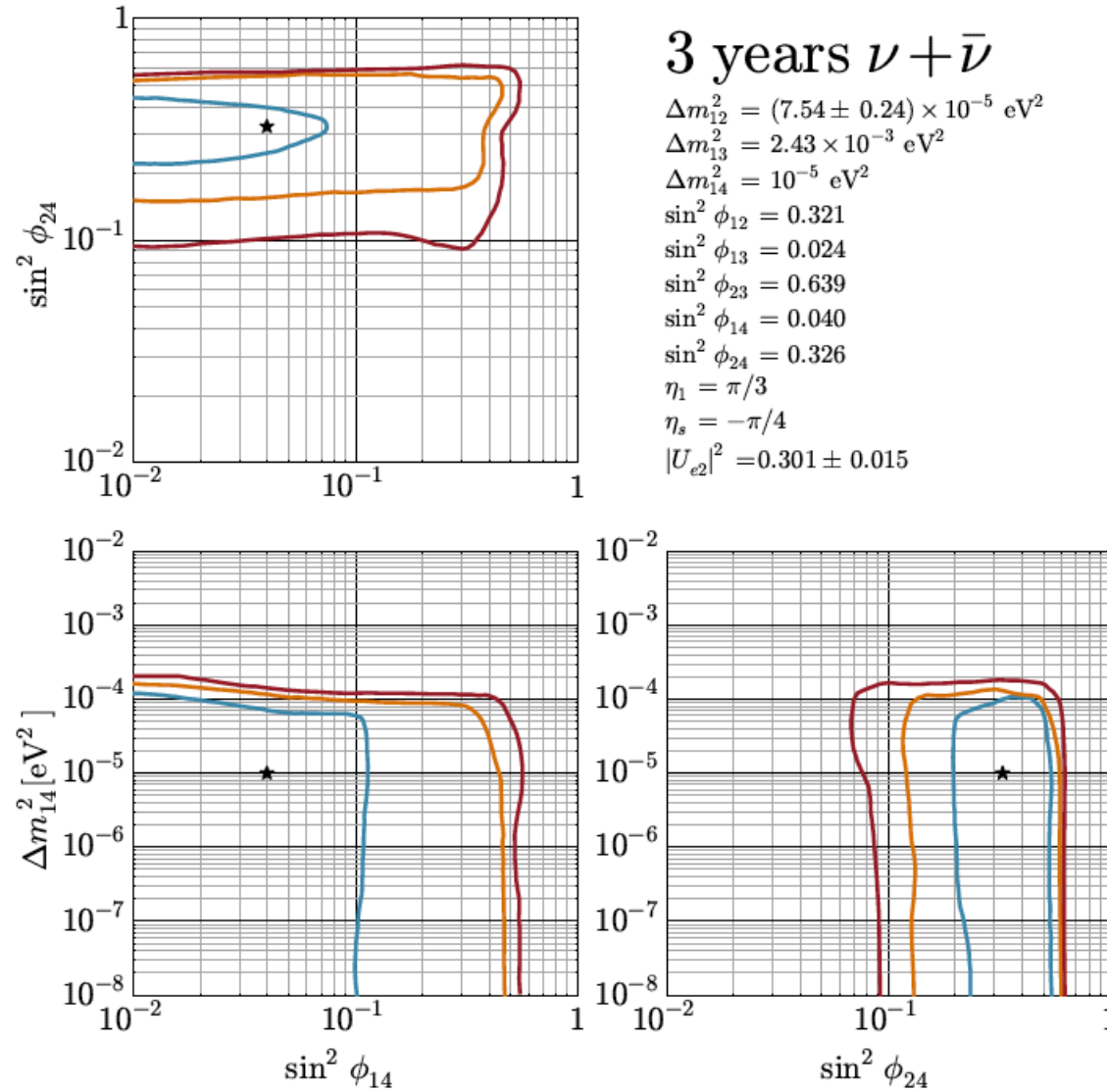


FIG. 5: Expected sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) CL at DUNE with six years of data collection (3y ν + 3y $\bar{\nu}$), a 34 kiloton detector, and a 1.2 MW beam given the existence of a fourth neutrino with parameters from Case 2 in Table I. Results from solar neutrino experiments are included here as Gaussian priors for the values of $|U_{e2}|^2 = 0.301 \pm 0.015$ and $\Delta m_{12}^2 = 7.54 \pm 0.24 \times 10^{-5} \text{ eV}^2$ [22].

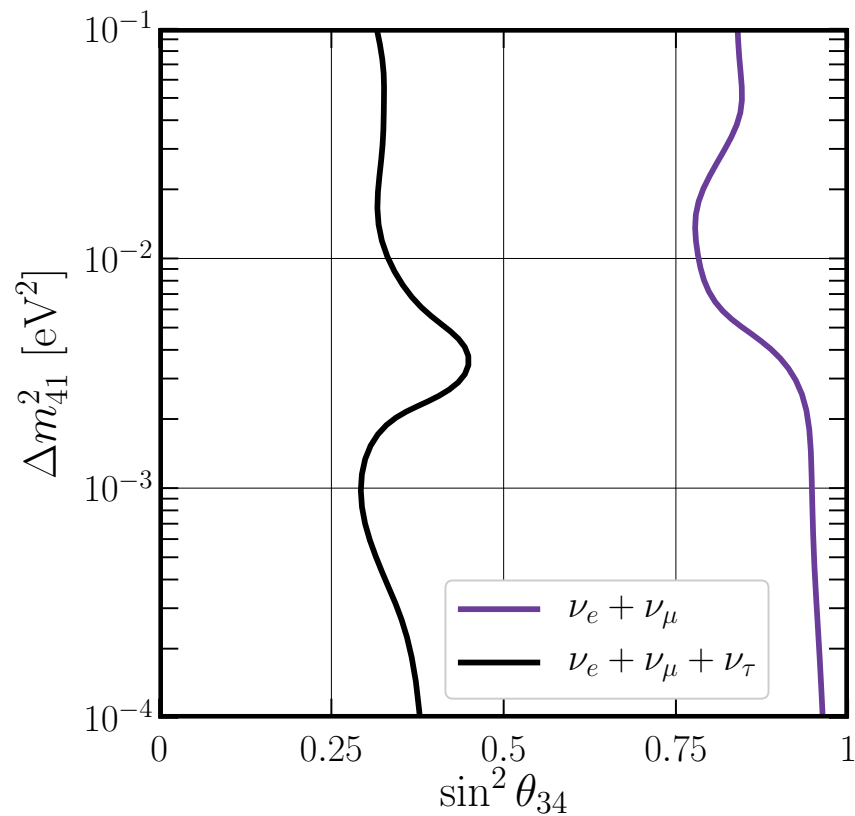
	$\sin^2 \phi_{14}$	$\sin^2 \phi_{24}$	$\Delta m_{14}^2 \text{ (eV}^2\text{)}$	η_s	$\sin^2 \phi_{12}$	$\sin^2 \phi_{13}$	$\sin^2 \phi_{23}$	$\Delta m_{12}^2 \text{ (eV}^2\text{)}$	$\Delta m_{13}^2 \text{ (eV}^2\text{)}$	η_1
Case 1	0.023	0.030	0.93	$-\pi/4$	0.315	0.0238	0.456	7.54×10^{-5}	2.43×10^{-3}	$\pi/3$
Case 2	0.023	0.030	1.0×10^{-2}	$-\pi/4$	0.315	0.0238	0.456	7.54×10^{-5}	2.43×10^{-3}	$\pi/3$
Case 3	0.040	0.320	1.0×10^{-5}	$-\pi/4$	0.321	0.0244	0.639	7.54×10^{-5}	2.43×10^{-3}	$\pi/3$

TABLE I: Input values of the parameters for the three scenarios considered for the four-neutrino hypothesis. Values of ϕ_{12} , ϕ_{13} , and ϕ_{23} are chosen to be consistent with the best-fit values of $|U_{e2}|^2$, $|U_{e3}|^2$, and $|U_{\mu 3}|^2$, given choices of ϕ_{14} and ϕ_{24} . Here, $\eta_s \equiv \eta_2 - \eta_3$. Note that Δm_{14}^2 is explicitly assumed to be positive, i.e., $m_4^2 > m_1^2$.



[Berryman et al, arXiv:1507.03986]

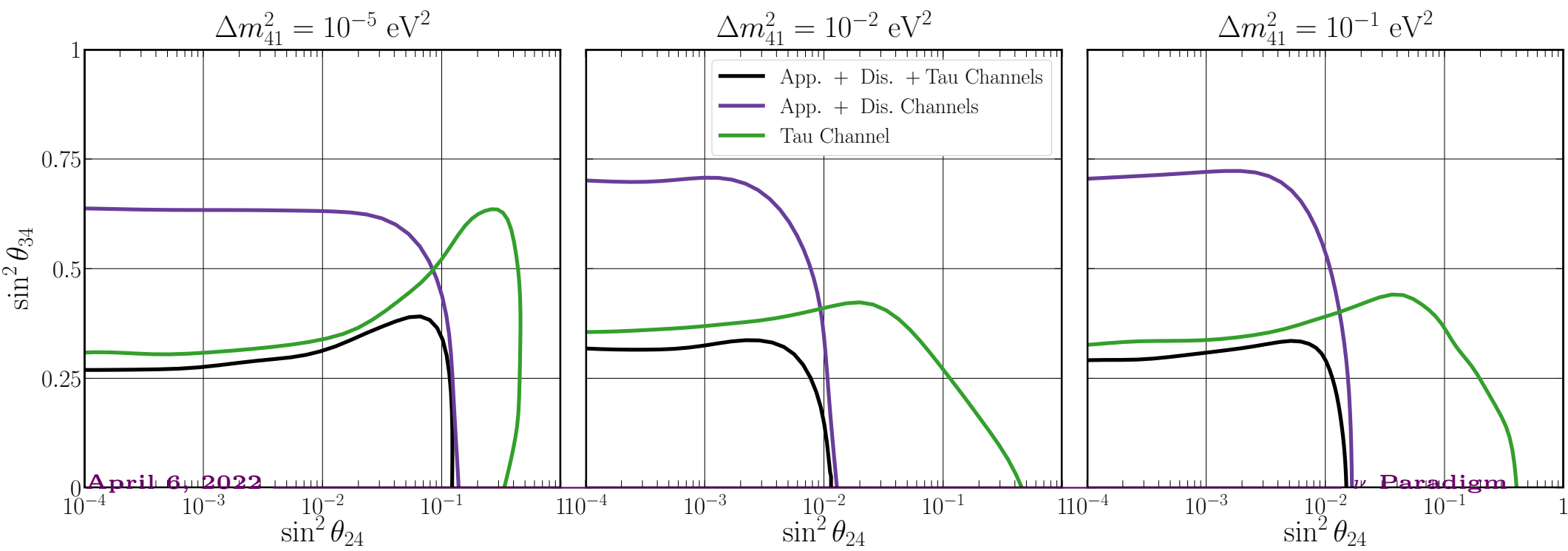
FIG. 6: Expected sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) CL at DUNE with six years of data collection (3y $\nu + 3\bar{\nu}$), a 34 kiloton detector, and a 1.2 MW beam given the existence of a fourth neutrino with parameters from Case 3 in Table I. Results from solar neutrino experiments are included here as Gaussian priors for the values of $|U_{e2}|^2 = 0.301 \pm 0.015$ and $\Delta m_{12}^2 = 7.54 \pm 0.24 \times 10^{-5} \text{ eV}^2$ [22].

A

Northwestern

Fourth Neutrino Hypothesis

[AdG, Kelly, Pasquini, Stenico, arXiv:1904.07265]



April 6, 2022

Paradigm

Non-Standard Neutrino Interactions (NSI)

Effective Lagrangian:

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F(\bar{\nu}_\alpha\gamma_\rho\nu_\beta) \sum_{f=e,u,d} (\epsilon_{\alpha\beta}^{fL}\bar{f}_L\gamma^\rho f_L + \epsilon_{\alpha\beta}^{fR}\bar{f}_R\gamma^\rho f_R) + h.c.,$$

For oscillations,

$$H_{ij} = \frac{1}{2E_\nu} \text{diag} \{0, \Delta m_{12}^2, \Delta m_{13}^2\} + V_{ij},$$

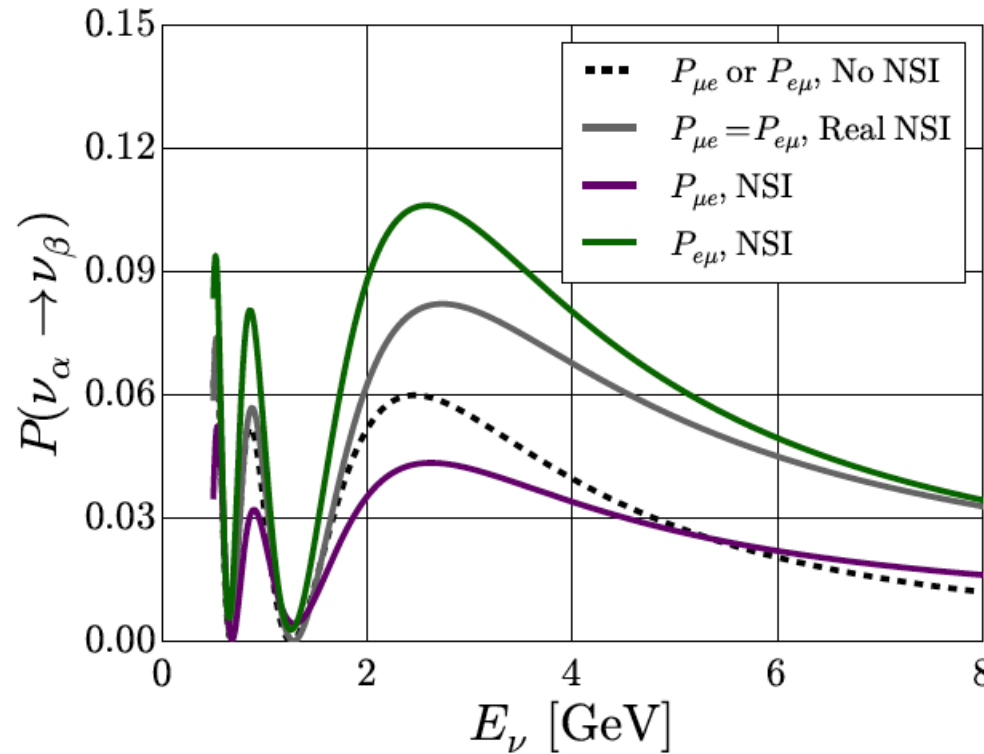
where

$$V_{ij} = U_{i\alpha}^\dagger V_{\alpha\beta} U_{\beta j},$$

$$V_{\alpha\beta} = A \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix},$$

$A = \sqrt{2}G_F n_e$. $\epsilon_{\alpha\beta}$ are linear combinations of the $\epsilon_{\alpha\beta}^{fL,R}$. Important: I will discuss propagation effects only and ignore NSI effects in production or detection (ϵ versus ϵ^2).

There are new sources of CP-invariance violation! [easier to see T-invariance violation]



[AdG and Kelly, arXiv:1511.05562]

FIG. 2: T -invariance violating effects of NSI at $L = 1300$ km for $\epsilon_{e\mu} = 0.1e^{i\pi/3}$, $\epsilon_{e\tau} = 0.1e^{-i\pi/4}$, $\epsilon_{\mu\tau} = 0.1$ (all other NSI parameters are set to zero). Here, the three-neutrino oscillation parameters are $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{13} = 0.0234$, $\sin^2 \theta_{23} = 0.437$, $\Delta m_{12}^2 = 7.54 \times 10^{-5} \text{ eV}^2$, $\Delta m_{13}^2 = 2.47 \times 10^{-3} \text{ eV}^2$, and $\delta = 0$, i.e., no “standard” T -invariance violation. The green curve corresponds to $P_{e\mu}$ while the purple curve corresponds to $P_{\mu e}$. If, instead, all non-zero NSI are real ($\epsilon_{e\mu} = 0.1$, $\epsilon_{e\tau} = 0.1$, $\epsilon_{\mu\tau} = 0.1$), $P_{e\mu} = P_{\mu e}$, the grey curve. The dashed line corresponds to the pure three-neutrino oscillation probabilities assuming no T -invariance violation (all $\epsilon_{\alpha\beta} = 0$, $\delta = 0$).

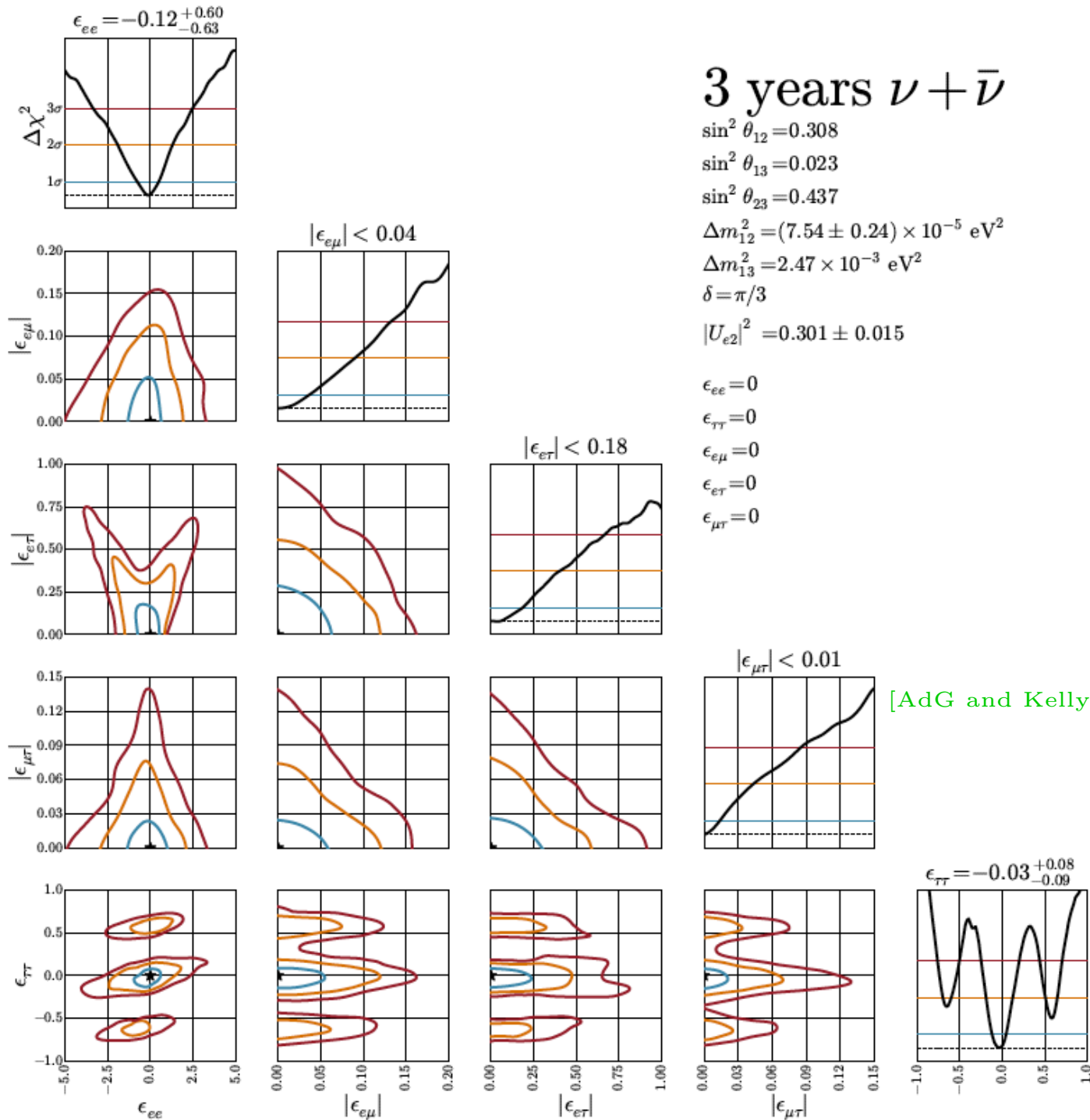
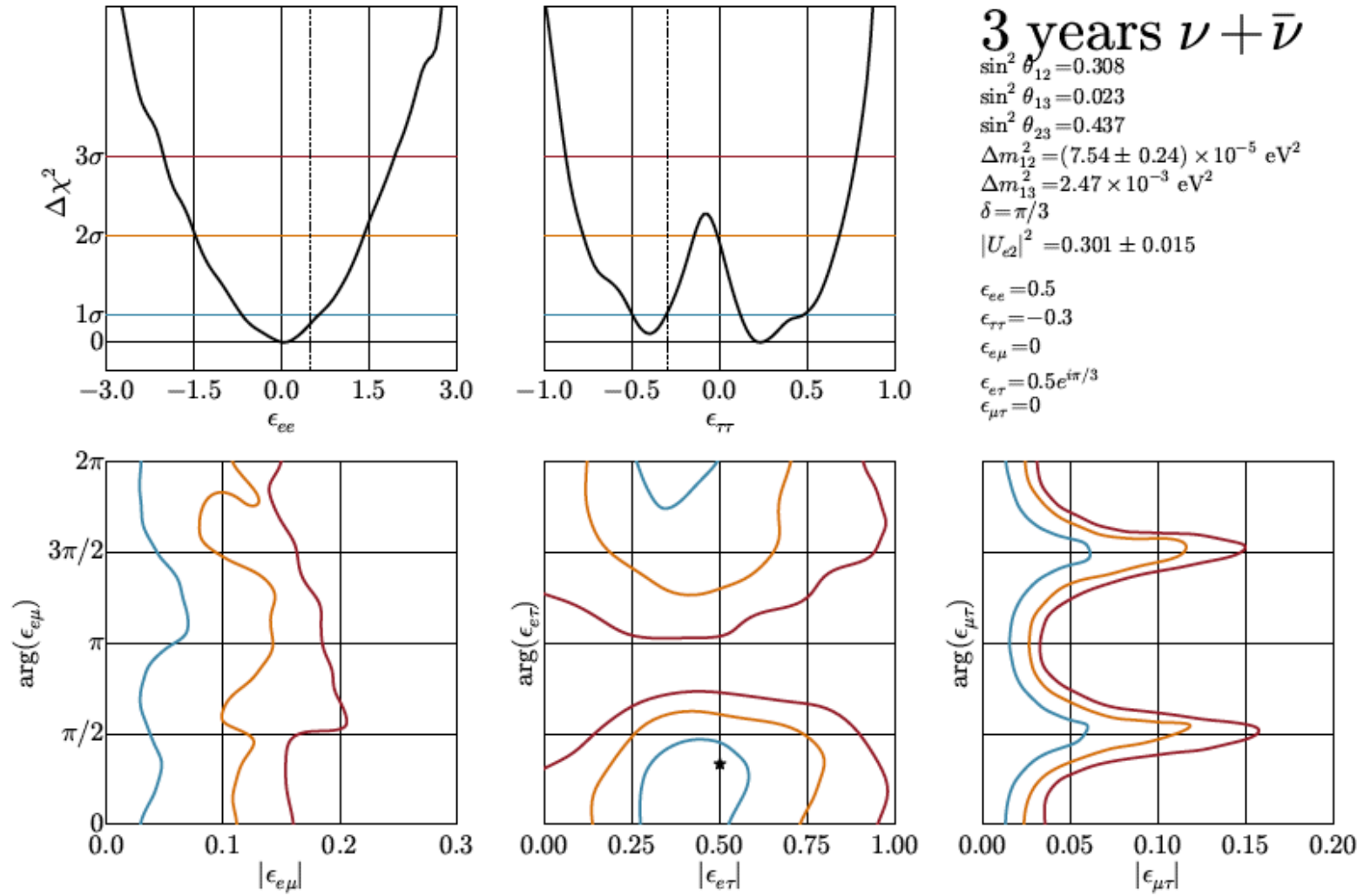


FIG. 4: Expected exclusion limits at 68.3% (red), 95% (orange), and 99% (blue) CL at DUNE assuming data consistent with the standard paradigm. The CP violation phase is fixed to $\delta = \pi/2$ and the ordering is normal. The ϵ_{ee} and $\epsilon_{\tau\tau}$ are fixed to 0.



	ϵ_{ee}	$\epsilon_{e\mu}$	$\epsilon_{e\tau}$	$\epsilon_{\mu\mu}^*$	$\epsilon_{\mu\tau}$	$\epsilon_{\tau\tau}$
Case 1	0	$0.15e^{i\pi/3}$	$0.3e^{-i\pi/4}$	0	0.05	0
Case 2	-1.0	0	0	0	0	0.3
Case 3	0.5	0	$0.5e^{i\pi/3}$	0	0	-0.3

[AdG and Kelly, arXiv:1511.05562]

TABLE I: Input values of the new physics parameters for the three NSI scenarios under consideration. The star symbol is a reminder that, as discussed in the text, we can choose $\epsilon_{\mu\mu} \equiv 0$ and reinterpret the other diagonal NSI parameters.

Telling Different Scenarios Apart:

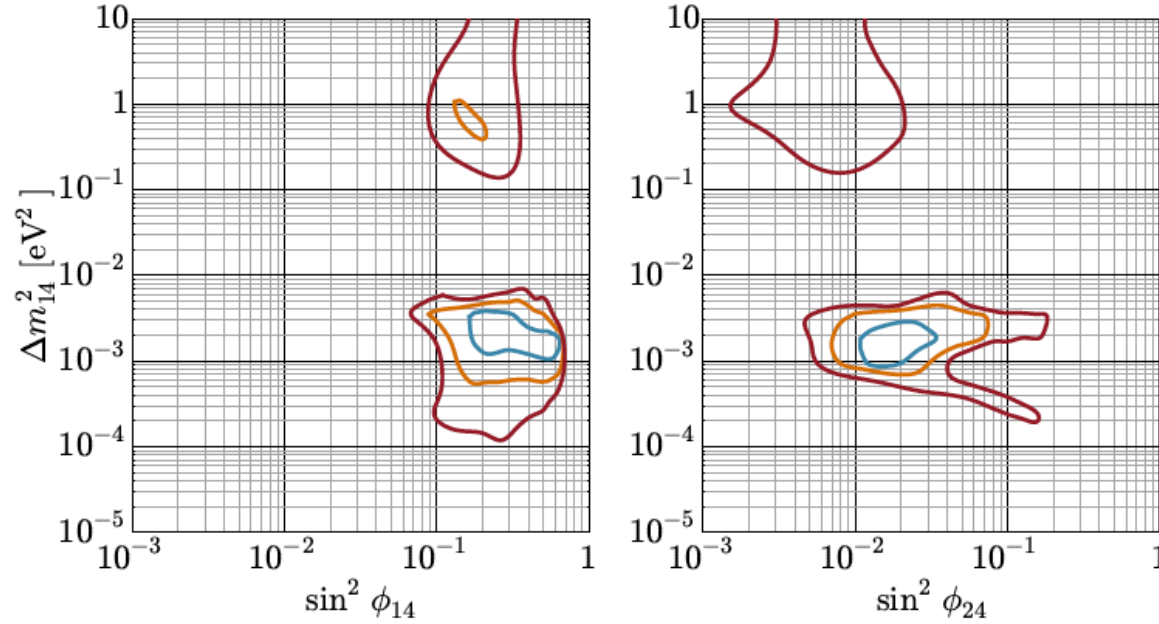


FIG. 8: Sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) for a four-neutrino fit to data consistent with Case 2 from Table I. All unseen parameters are marginalized over, and Gaussian priors are included on the values of Δm_{12}^2 and $|U_{e2}|^2$. See text for details.

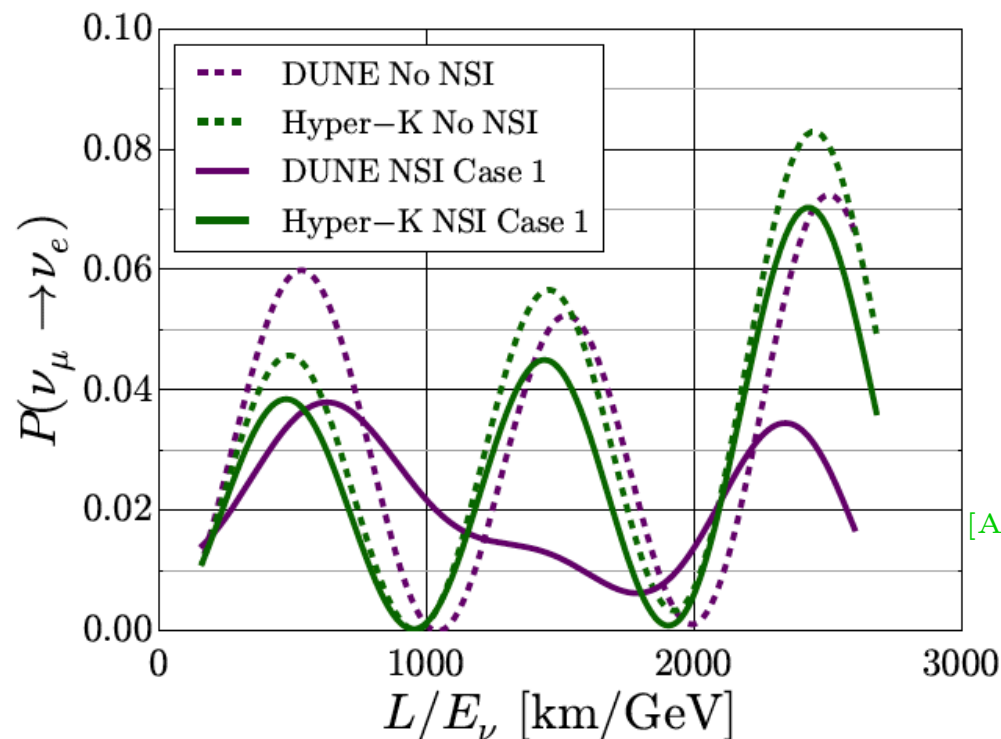
[AdG and Kelly, arXiv:1511.05562]

Fit	Case 1	Case 2	Case 3
3ν with Solar Priors	217/114 $\simeq 5.4\sigma$	186/114 $\simeq 4.2\sigma$	118/114 $\simeq 4.3\sigma$
3ν without Priors	172/114 $\simeq 3.4\sigma$	134/114 $\simeq 1.6\sigma$	154/114 $\simeq 2.7\sigma$
4ν with Solar Priors	193/110 $\simeq 4.8\sigma$	142/110 $\simeq 2.3\sigma$	153/110 $\simeq 2.8\sigma$

TABLE II: Results of various three- or four-neutrino fits to data generated to be consistent with the cases listed in Table I. Numbers quoted are for χ^2_{\min}/dof and the equivalent discrepancy using a χ^2 distribution.

How Do We Learn More – Different Experiments!

- Different L and E , same L/E (e.g. HyperK or ESSnuSB versus DUNE);
- Different matter potentials (e.g. atmosphere versus accelerator);
- Different oscillation modes (appearance versus disappearance, e 's, μ 's and τ 's).



[AdG and Kelly, arXiv:1511.05562]

FIG. 9: Oscillation probabilities for three-neutrino (dashed) and NSI (solid) hypotheses as a function of L/E_ν , the baseline length divided by neutrino energy, for the DUNE (purple) and HyperK (green) experiments. Here, $\delta = 0$ and the three-neutrino parameters used are consistent with Ref. [47].

Solar Neutrinos

We are not done yet!

- see “vacuum-matter” transition
- probe for new physics: NSI, pseudo-Dirac, ...
- probe of the solar interior! “solar abundance problem” (see e.g. 1104.1639)
- ‘CNO neutrinos may provide information on planet formation!’

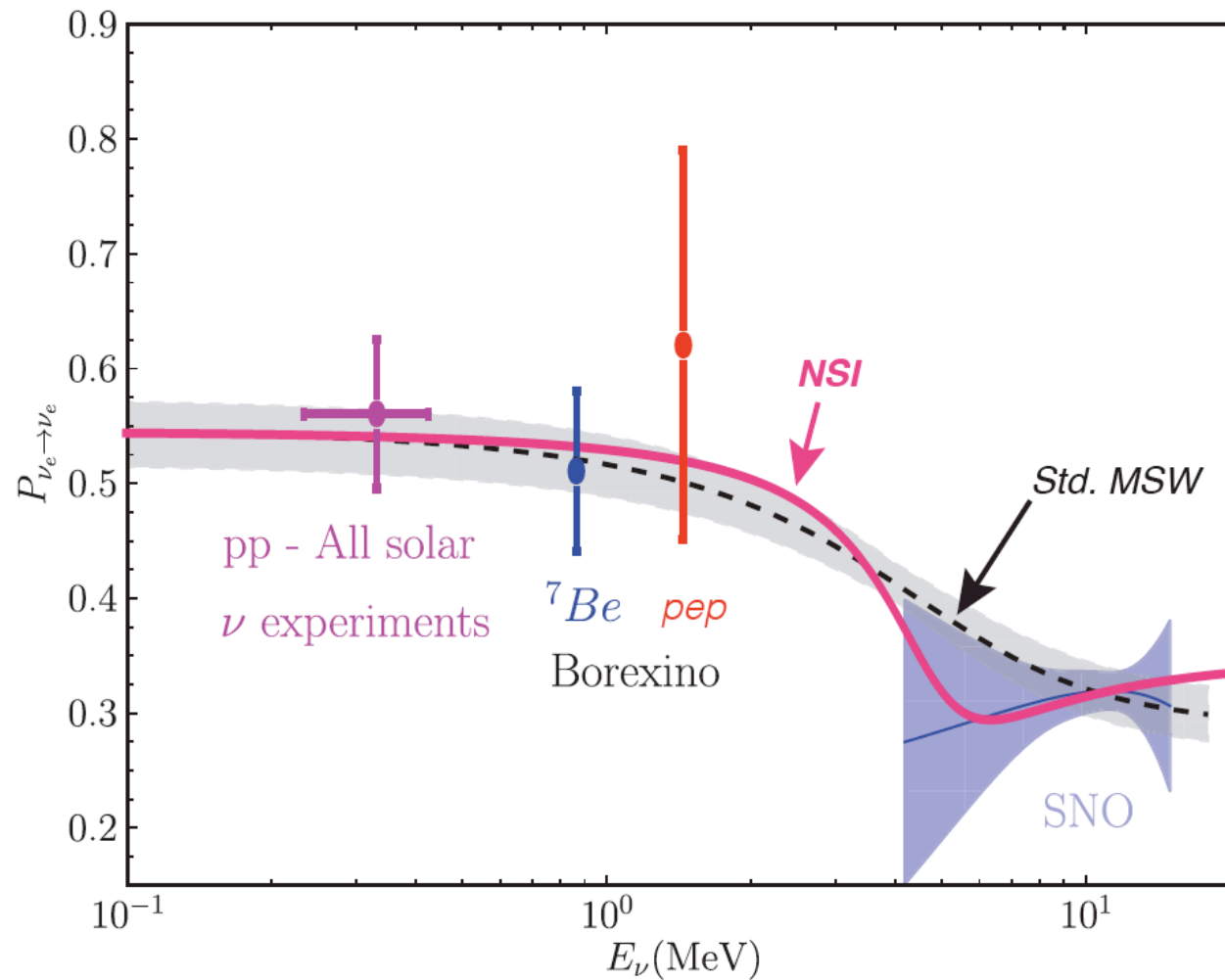


FIG. 1: Recent SNO solar neutrino data [18] on $P(\nu_e \rightarrow \nu_e)$ (blue line with 1σ band). The LMA MSW solution (dashed black curve with gray 1σ band) appears divergent around a few MeV, whereas for NSI with $\epsilon_{e\tau} = 0.4$ (thick magenta), the electron neutrino probability appears to fit the data better. The data points come from the recent Borexino paper [19].

[Friedland, Shoemaker 1207.6642]

OSC			+COHERENT		
	LMA	LMA \oplus LMA-D		LMA	LMA \oplus LMA-D
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	$[-0.020, +0.456]$	$\oplus[-1.192, -0.802]$	ε_{ee}^u	$[-0.008, +0.618]$	$[-0.008, +0.618]$
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	$[-0.005, +0.130]$	$[-0.152, +0.130]$	$\varepsilon_{\mu\mu}^u$	$[-0.111, +0.402]$	$[-0.111, +0.402]$
$\varepsilon_{e\mu}^u$	$[-0.060, +0.049]$	$[-0.060, +0.067]$	$\varepsilon_{\tau\tau}^u$	$[-0.110, +0.404]$	$[-0.110, +0.404]$
$\varepsilon_{e\tau}^u$	$[-0.292, +0.119]$	$[-0.292, +0.336]$	$\varepsilon_{e\mu}^u$	$[-0.060, +0.049]$	$[-0.060, +0.049]$
$\varepsilon_{\mu\tau}^u$	$[-0.013, +0.010]$	$[-0.013, +0.014]$	$\varepsilon_{e\tau}^u$	$[-0.248, +0.116]$	$[-0.248, +0.116]$
$\varepsilon_{ee}^d - \varepsilon_{\mu\mu}^d$	$[-0.027, +0.474]$	$\oplus[-1.232, -1.111]$	$\varepsilon_{\mu\tau}^u$	$[-0.012, +0.009]$	$[-0.012, +0.009]$
$\varepsilon_{\tau\tau}^d - \varepsilon_{\mu\mu}^d$	$[-0.005, +0.095]$	$[-0.013, +0.095]$	ε_{ee}^d	$[-0.012, +0.565]$	$[-0.012, +0.565]$
$\varepsilon_{e\mu}^d$	$[-0.061, +0.049]$	$[-0.061, +0.073]$	$\varepsilon_{\mu\mu}^d$	$[-0.103, +0.361]$	$[-0.103, +0.361]$
$\varepsilon_{e\tau}^d$	$[-0.247, +0.119]$	$[-0.247, +0.119]$	$\varepsilon_{\tau\tau}^d$	$[-0.102, +0.361]$	$[-0.102, +0.361]$
$\varepsilon_{\mu\tau}^d$	$[-0.012, +0.009]$	$[-0.012, +0.009]$	$\varepsilon_{e\mu}^d$	$[-0.058, +0.049]$	$[-0.058, +0.049]$
$\varepsilon_{ee}^p - \varepsilon_{\mu\mu}^p$	$[-0.041, +1.312]$	$\oplus[-3.328, -1.958]$	$\varepsilon_{e\tau}^d$	$[-0.206, +0.110]$	$[-0.206, +0.110]$
$\varepsilon_{\tau\tau}^p - \varepsilon_{\mu\mu}^p$	$[-0.015, +0.426]$	$[-0.424, +0.426]$	$\varepsilon_{\mu\tau}^d$	$[-0.011, +0.009]$	$[-0.011, +0.009]$
$\varepsilon_{e\mu}^p$	$[-0.178, +0.147]$	$[-0.178, +0.178]$	ε_{ee}^p	$[-0.010, +2.039]$	$[-0.010, +2.039]$
$\varepsilon_{e\tau}^p$	$[-0.954, +0.356]$	$[-0.954, +0.949]$	$\varepsilon_{\mu\mu}^p$	$[-0.364, +1.387]$	$[-0.364, +1.387]$
$\varepsilon_{\mu\tau}^p$	$[-0.035, +0.027]$	$[-0.035, +0.035]$	$\varepsilon_{\tau\tau}^p$	$[-0.350, +1.400]$	$[-0.350, +1.400]$
			$\varepsilon_{e\mu}^p$	$[-0.179, +0.146]$	$[-0.179, +0.146]$
			$\varepsilon_{e\tau}^p$	$[-0.860, +0.350]$	$[-0.860, +0.350]$
			$\varepsilon_{\mu\tau}^p$	$[-0.035, +0.028]$	$[-0.035, +0.028]$

Table 1. 2σ allowed ranges for the NSI couplings $\varepsilon_{\alpha\beta}^u$, $\varepsilon_{\alpha\beta}^d$ and $\varepsilon_{\alpha\beta}^p$ as obtained from the global analysis of oscillation data (left column) and also including COHERENT constraints. The results are obtained after marginalizing over oscillation and the other matter potential parameters either within the LMA only and within both LMA and LMA-D subspaces respectively (this second case is denoted as LMA \oplus LMA-D).

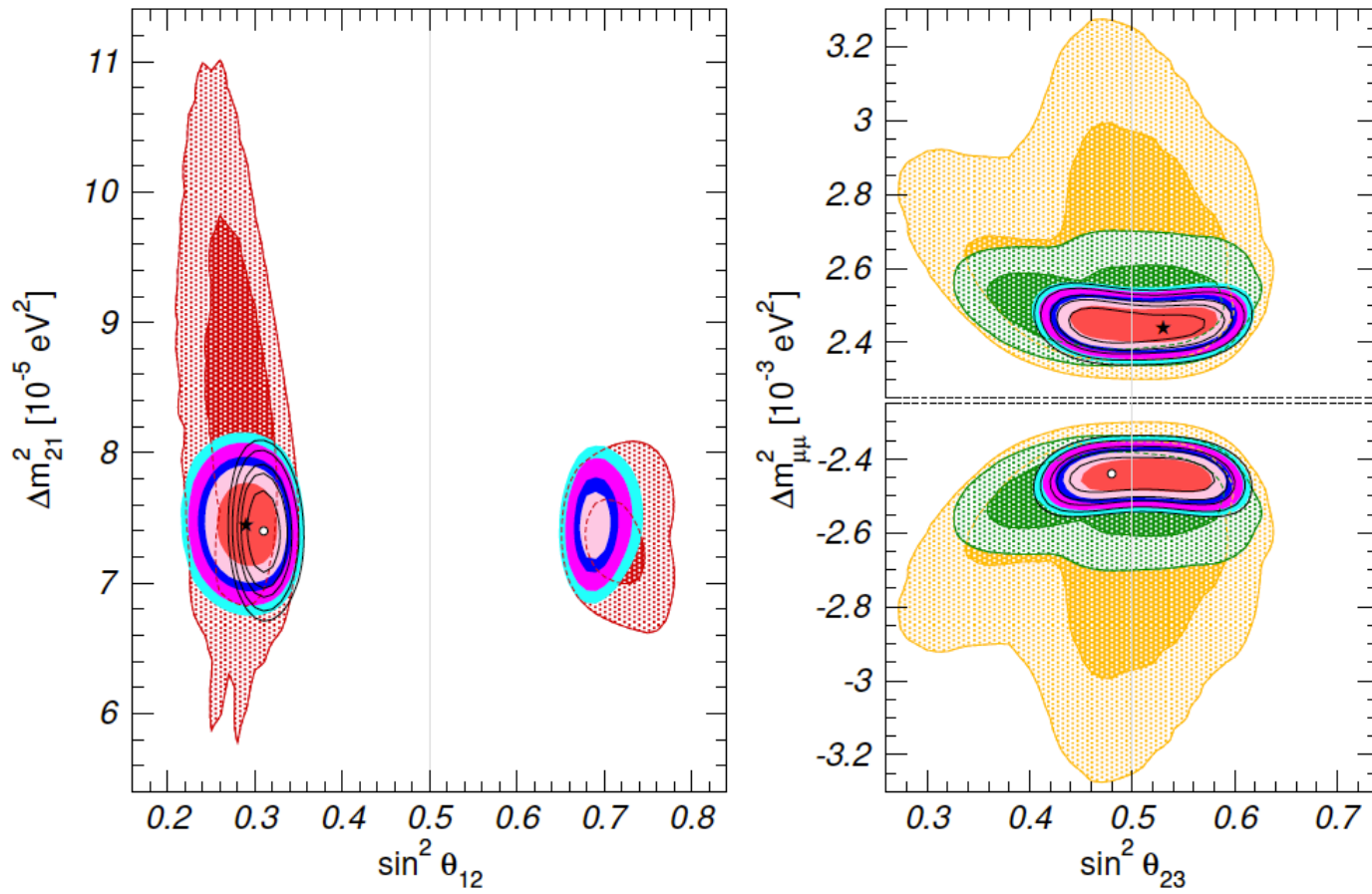


Figure 6. Two-dimensional projections of the allowed regions onto different vacuum parameters after marginalizing over the matter potential parameters (including η) and the undisplayed oscillation parameters. The solid colored regions correspond to the global analysis of all oscillation data, and show the 1σ , 90%, 2σ , 99% and 3σ CL allowed regions; the best-fit point is marked with a star. The black void regions correspond to the analysis with the standard matter potential (*i.e.*, without NSI) and its best-fit point is marked with an empty dot. For comparison, in the left panel we show in red the 90% and 3σ allowed regions including only solar and KamLAND results, while in the right panels we show in green the 90% and 3σ allowed regions excluding solar and KamLAND data, and in yellow the corresponding ones excluding also IceCube and reactor data.

The Physics Behind NSI – Comments and Concerns

There are two main questions associated to NSI's. They are somewhat entwined.

1. What is the new physics that leads to neutrino NSI? or are there models for new physics that lead to large NSIs? Are these models well motivated? Are they related to some of the big questions in particle physics?
2. Are NSIs constrained by observables that have nothing to do with neutrino physics? Are large NSI effects allowed at all?

Effective Lagrangian:

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\epsilon^{\alpha\beta}(\bar{\nu}_\alpha\gamma_\rho\nu_\beta)(\bar{f}\gamma^\rho f).$$

This is not $SU(2)_L$ invariant. Let us fix that:

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\epsilon^{\alpha\beta}(\bar{L}_\alpha\gamma_\rho L_\beta)(\bar{f}\gamma^\rho f).$$

where $L = (\nu, \ell^-)^T$ is the lepton doublet. This is a big problem.

Charged-Lepton flavor violating constraints are really strong (think $\mu \rightarrow e^+e^-e^+$, $\mu \rightarrow e$ -conversion, $\tau \rightarrow \mu$ +hadrons, etc), and so are most of the flavor diagonal charged-lepton effects.

There are a couple of ways to circumvent this...

1. Dimension-Eight Effective Operator

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\epsilon^{\alpha\beta}(\bar{\nu}_\alpha\gamma_\rho\nu_\beta)(\bar{f}\gamma^\rho f).$$

This is not $SU(2)_L$ invariant. Let us fix that **in a different way**

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\frac{\epsilon^{\alpha\beta}}{v^2}((HL)_\alpha^\dagger\gamma_\rho(HL)_\beta)(\bar{f}\gamma^\rho f).$$

where $HL \propto H^+\ell^- - H^0\nu$. After electroweak symmetry breaking $H^0 \rightarrow v + h^0$ and we only get new neutrino interactions.

Sadly, it is not that simple. At the one-loop level, the dimension-8 operator will contribute to the dimension-6 operator in the last page, as discussed in detail in [Gavela *et al*, arXiv:0809.3451 [hep-ph]]. One can, however, fine-tune away the charged-lepton effects.

2. Light Mediator

(Overview by Y. Farzan and M. Tórtola, [arXiv:1710.09360 \[hep-ph\]](#))

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\epsilon^{\alpha\beta}(\bar{\nu}_\alpha\gamma_\rho\nu_\beta)(\bar{f}\gamma^\rho f).$$

This may turn out to be a good effective theory for neutrino propagation but a bad effective theory for most charged-lepton processes. I.e.

$$\mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F\epsilon^{\alpha\beta}(\bar{L}_\alpha\gamma_\rho L_\beta)(\bar{f}\gamma^\rho f).$$

might be inappropriate for describing charged-lepton processes if the particle we are integrating out is light (as in lighter than the muon).

Charged-lepton processes are “watered down.” Very roughly

$$\epsilon \rightarrow \epsilon \left(\frac{m_{Z'}}{m_\ell} \right)^2$$

where $m_{Z'}$ is the mass of the particle mediating the new interaction, and m_ℓ is the mass associated to the charged-lepton process of interest.